Transport and dissipation in neutron star mergers

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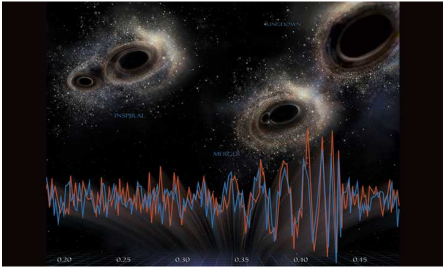
Outline

- Neutron star mergers
- Thermal conductivity
- Bulk viscosity
- Conclusions

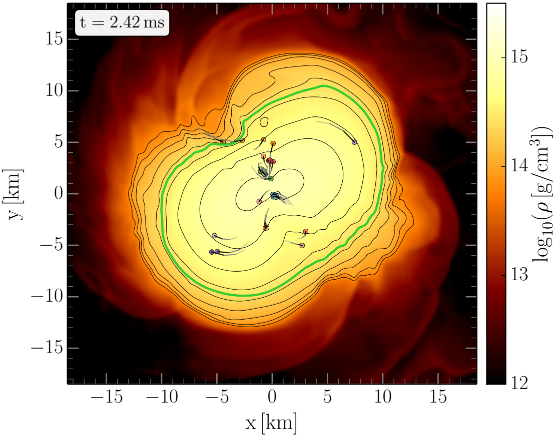
Neutron star mergers



LIGO has seen black hole mergers and one neutron star merger.

- We expect to observe many neutron star mergers in the coming years
- If we can achieve a quantitative understanding of neutron star mergers, perhaps we could use their gravitational wave, electromagentic, and neutrino signals to learn about properties of nuclear matter under extreme conditions

Nuclear material in a neutron star merger



Density:
up to 4 n_{sat}

Temperature:
5 to 20 MeV

Significant spatial/temporal variation in:
- temperature
- fluid flow velocity
- density

so we need to allow for
- thermal conductivity
- shear viscosity
- bulk viscosity

Summary

It is useful to have estimates of the equilibration times for various forms of dissipation, to decide which is the most important.

- Thermal equilibration: If neutrinos are trapped, and there are short-distance temperature gradients then thermal transport might be fast enough to play a role.

$$\tau_{\kappa}^{(\nu)} \approx 700\text{ms} \left(\frac{z_{\mathrm{typ}}}{1\,\mathrm{km}}\right)^{2} \left(\frac{T}{10\,\mathrm{MeV}}\right)^{2} \left(\frac{0.1}{x_p}\right)^{1/3} \left(\frac{m_n^*}{0.8\,m_n}\right)^{3} \left(\frac{\mu_e}{2\mu_\nu}\right)^{2}$$

- Shear viscosity: similar conclusion.
- Bulk viscosity: If Direct Urca processes remain suppressed at the relevant densities and temperatures, bulk viscosity will quickly damp density oscillations

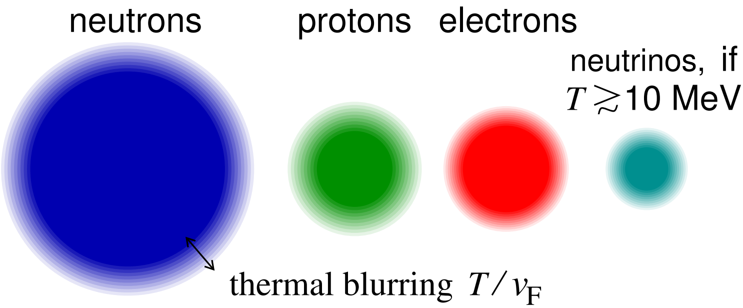
$$\tau_{\zeta}^{\mathrm{min}} \approx 3\,\text{ms} \left(\frac{t_{\mathrm{comp}}}{1\,\mathrm{ms}}\right) \left(\frac{K}{250\,\text{MeV}}\right) \left(\frac{0.25\,\mathrm{MeV}}{Y_\zeta}\right)$$

Nuclear material constituents

Temperature: 5 to 20 MeV

Density: up to 4 n_{sat}

Fermi surfaces:



neutrons

protons

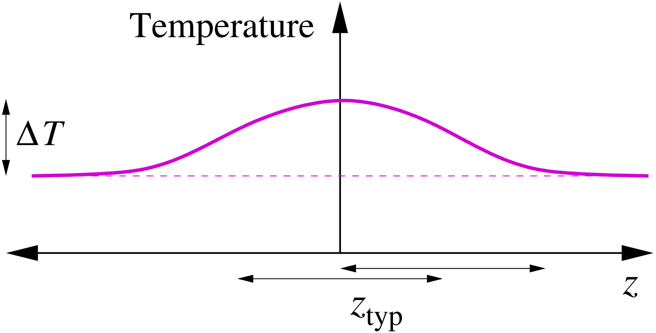
electrons

neutrinos, if $T \gtrsim 10$ MeV

thermal blurring T / v_{F}

neutrons:	$\sim 90\%$ of baryons	$p_{Fn} \sim 350\,\mathrm{MeV}$
protons:	$\sim 10\%$ of baryons	$p_{Fp} \sim 150\,\mathrm{MeV}$
electrons:	same density as protons	$p_{Fe} = p_{Fp}$
neutrinos:	only present if mfp $\lesssim 1\,\mathrm{km}$	$p_{F\nu} \sim \frac{1}{2} p_{Fe}$

Thermal equilibration



Volume

$V \sim z_{\text{typ}}^3$

Surface area

$A \sim 6z_{\text{typ}}^2$

Extra heat in region: $E_{\text{therm}} = c_V V \Delta T \approx c_V z_{\text{typ}}^3 \Delta T$

Rate of heat outflow: $W_{\text{therm}} = \kappa \dfrac{dT}{dz} A \approx \kappa \dfrac{\Delta T}{z_{\text{typ}}} 6z_{\text{typ}}^2$

$$\boxed{\text{Time to equilibrate:} \quad \tau_\kappa = \frac{E_{\text{therm}}}{W_{\text{therm}}} \approx \frac{c_V z_{\text{typ}}^2}{6\kappa}}$$

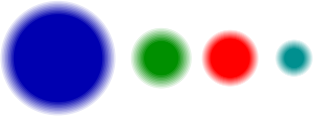
Thermal equilibration time

$$\boxed{\text{Time to equilibrate:}\quad \tau_\kappa = \frac{E_{\rm therm}}{W_{\rm therm}} \approx \frac{c_V z_{\rm typ}^2}{6\kappa}}$$

In neutron star mergers, things happen on the 10 ms timescale.

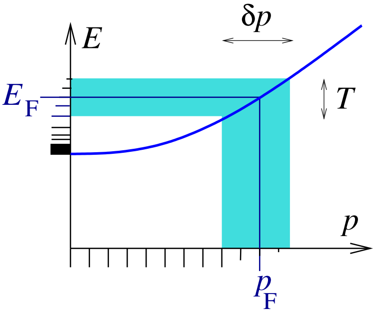
Thermal diffusion is important if $\tau_\kappa \lesssim 10\,\mathrm{ms}$

Specific heat capacity:



Dominated by neutrons

$$\begin{aligned} c_V &\sim \text{number of states available} \\ &\quad\ \text{to carry energy} \lesssim T \\ &\sim \text{vol of mom space with states available to carry energy} \lesssim T \\ &\sim {p_{Fn}}^2 \delta p \end{aligned}$$



$$\delta p = \frac{T}{v_{Fn}} = T \times \frac{m_n^*}{p_{Fn}}$$

$$c_V \sim {p_{Fn}}^2 \delta p \sim {p_{Fn}}^2 \frac{m_n^*}{p_{Fn}} T \sim m_n^* p_{Fn} T$$

(Note: neutron density $n_n \sim {p_{Fn}}^3$)

$$\boxed{c_V \approx 1.0\, m_n^*\, {n_n}^{1/3}\, T}$$

Thermal conductivity

Thermal conductivity $\kappa \propto n\, v\, \lambda$

Dominated by the species with the right combination of
- high density
- weak interactions $\Rightarrow$ long mean free path (mfp) λ

neutrons: high density, but strongly interacting (short mfp) ✗

protons: low density, strongly interacting (short mfp) ✗

electrons: low density, only E&M interactions (long mfp) ✓

neutrinos:
- $T \lesssim 10\,\mathrm{MeV}$: $\lambda >$ size of merged stars, so they all escape, density $= 0$ ✗
- $T \gtrsim 10\,\mathrm{MeV}$: $\lambda <$ size of merged stars, but still very long mfp! ✓✓

Electrons vs Neutrinos

$$\tau_\kappa \approx \frac{c_V z_{\rm typ}^2}{6\kappa}$$

electron-dominated ($T \lesssim 10\,\mathrm{MeV}$)

$$\kappa^{(e)} \approx 1.5\,\frac{n_e^{2/3}}{\alpha}$$

neutrino-dominated ($T \gtrsim 10\,\mathrm{MeV}$)

$$\kappa^{(\nu)} \approx 0.33\frac{n_\nu^{2/3}}{G_F^2 m_n^{*2} n_e^{1/3}\, T}$$

Equilibration time:

$$\tau_\kappa^{(e)} = \boxed{5\times 10^8\,\mathrm{s}} \left(\frac{z_{\rm typ}}{1\,\mathrm{km}}\right)^2 \left(\frac{T}{1\,\mathrm{MeV}}\right) \times \left(\frac{m_n^*}{0.8\,m_n}\right)\left(\frac{n_0}{n_n}\right)^{1/3}\left(\frac{0.1}{x_p}\right)^{2/3}$$

$$\tau_\kappa^{(\nu)} \approx \boxed{0.7\,\mathrm{s}} \left(\frac{z_{\rm typ}}{1\,\mathrm{km}}\right)^2 \left(\frac{T}{10\,\mathrm{MeV}}\right)^2 \times \left(\frac{\mu_e}{2\mu_\nu}\right)^2\left(\frac{0.1}{x_p}\right)^{1/3}\left(\frac{m_n^*}{0.8\,m_n}\right)^3$$

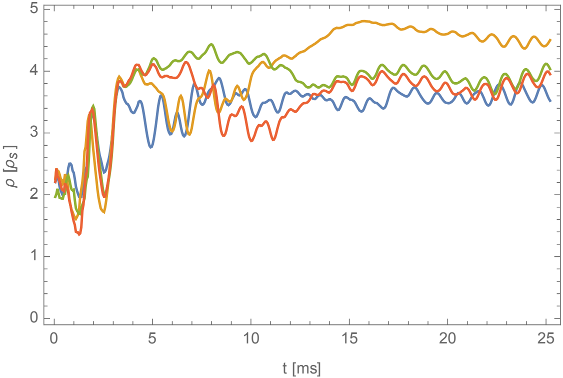
Electron thermal transport is *slow*!
electron mfp is too short

Neutrino thermal transport. . .
maybe if gradients on 0.1 km scale?

Bulk viscosity: compression dissipation

Bulk viscosity turns compression energy of density oscillations into heat.

Density vs time for tracers in merger



Tracers (co-moving fluid elements) show dramatic density oscillations especially in the first 5 ms.

Amplitude: up to 50%
Period: $\sim 2\,\mathrm{ms}$

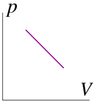
- What is the largest bulk viscosity $\zeta_{\max}$ we could expect?
- What is the equilibration time τ_ζ?
  Ie how long does it take for bulk viscosity to dissipate a good fraction of the energy of a density oscillation?

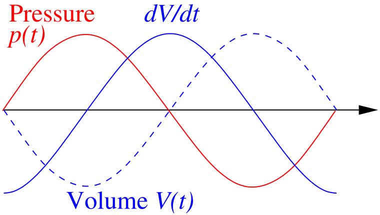
Bulk viscosity: phase lag in system response

Some component in the material is equilibrating slowly.
Baryon density n and hence fluid element volume V gets out of phase with applied pressure p:

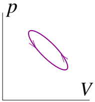
$$\text{Dissipation} = -\int p\,dV = -\int p\,\frac{dV}{dt}\,dt$$

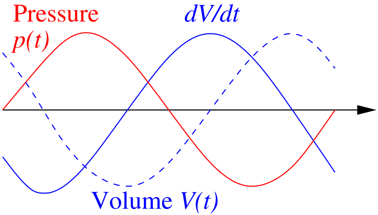
No phase lag.
Dissipation $= 0$





Some phase lag.
Dissipation > 0



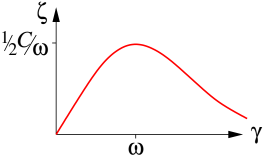


Bulk viscosity: a resonant phenomenon

Bulk viscosity is maximum when

$$\underset{\gamma}{\text{(internal equilibration rate)}} \sim \underset{\omega}{\text{(freq of density oscillation)}}$$

$$\zeta = C \frac{\gamma}{\gamma^2 + \omega^2}$$



What quantity would equilibrate on the timescale of the density oscillations in neutron star mergers (milliseconds)?

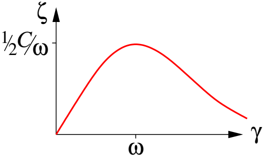
Bulk viscosity: a resonant phenomenon

Bulk viscosity is maximum when

(internal equilibration rate) $\sim$ (freq of density oscillation)

γ $\qquad$ ω

$$\zeta = C\frac{\gamma}{\gamma^2+\omega^2}$$



What quantity would equilibrate on the timescale of the density oscillations in neutron star mergers (milliseconds)?

Flavor, via weak interactions

Bulk viscosity and flavor equilibration

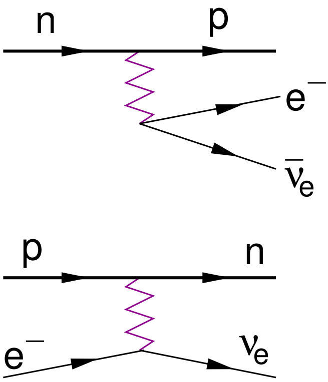
When you compress nuclear matter, the proton fraction wants to change.

Weak interactions convert $n \leftrightarrow p$

But exactly what is the timescale?
Is it similar to the millisecond timescale of density oscillations in neutron star mergers?

Flavor equilibration processes

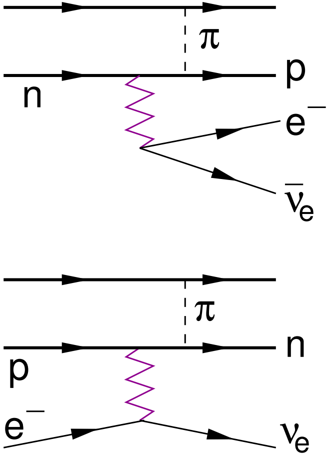
Direct Urca



Only occurs if proton density is high enough: $p_{Fn} < p_{Fe} + p_{Fp}$

Rate $\gamma_D \sim T^4$

Modified Urca



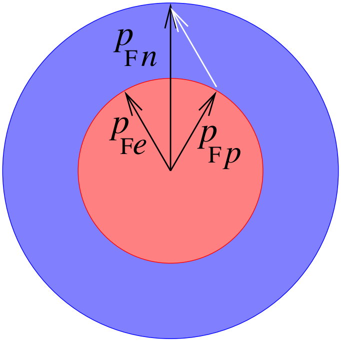
$\gamma_M \sim T^6$

When can Direct Urca happen?

$$n \to p\ e^-\ \bar{\nu}_e, \quad p\ e^- \to n\ \nu_e$$

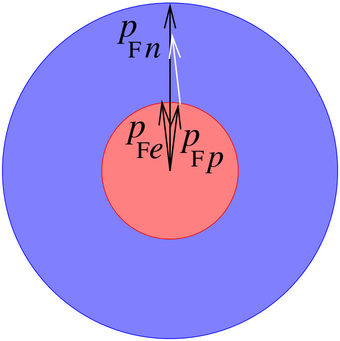
For $T = 0$ and the case of no neutrino trapping ($\mu_\nu = 0$)

High proton fraction:
Direct Urca open



$\vec{p}_n = \vec{p}_p + \vec{p}_e$ is possible
because $p_{Fn} < p_{Fp} + p_{Fe}$

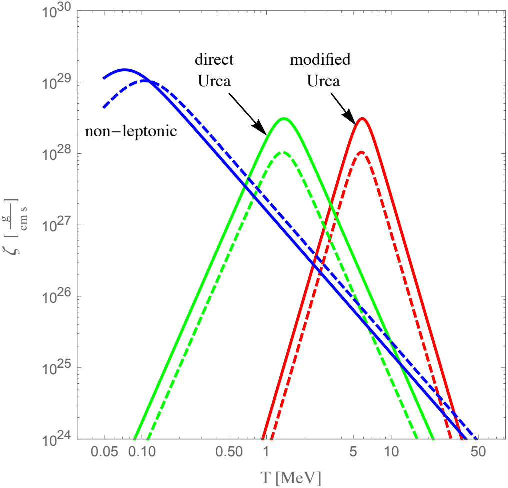
Low proton fraction:
Direct Urca closed



$\vec{p}_n = \vec{p}_p + \vec{p}_e$ is impossible
because $p_{Fn} > p_{Fp} + p_{Fe}$

Bulk viscosity: resonant peak

For oscillations of freq $\omega = 2\pi \times 1\,\mathrm{kHz}$



Bulk visc reaches maximum when flavor equilibration rate $\gamma(T) = \omega$.

Direct Urca is faster, so $\gamma_D(T) = \omega$ at $T \sim 1\,\mathrm{MeV}$ ζ suppressed at $T \gtrsim 5\,\mathrm{MeV}$

Modified Urca is slower, so $\gamma_M(T) = \omega$ at $T \sim 7\,\mathrm{MeV}$

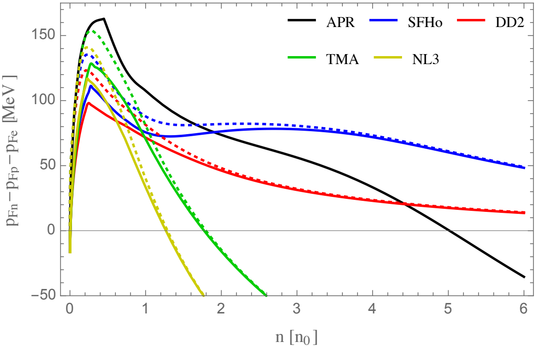
ζ_{max} is determined by EoS, indp of equilibration rate

Typical temperature in first 5ms of post-merger (where density oscillations are large) is 5-20 MeV, so we expect strong bulk viscosity if the Direct Urca channel is suppressed (proton density low).

Does Direct Urca occur?

Direct Urca happens when $p_{Fn} < p_{Fe} + p_{Fp}$

i.e. when $p_{Fn} - p_{Fe} - p_{Fp} < 0$



At $T = 0$, there is no consensus among candidate nuclear matter equations of state about the threshold density for Direct Urca.

Need to consider:

- Thermal effects
- Interaction effects
- Gradual opening of Direct Urca phase space
- Effects of ν_e trapping

The amount of bulk visc dissipation is a probe of the nuclear EoS

Bulk viscosity equilibration time

Density oscillation of amplitude Δn on timescale $t_{\rm comp}$:

$$n(t) = \bar{n} + \Delta n \cos(2\pi t / t_{\rm comp})$$

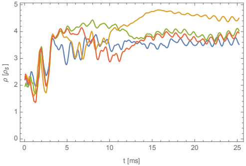
Energy of density oscillation: $\mathcal{E}_{\rm comp} = \dfrac{K}{18}\bar{n}\left(\dfrac{\Delta n}{\bar{n}}\right)^2$

Compression dissipation rate: $W_{\rm comp} = \dfrac{2\pi^2\zeta}{{t_{\rm comp}}^2}\left(\dfrac{\Delta n}{\bar{n}}\right)^2$

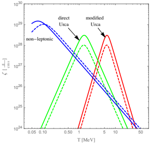
$$\boxed{\text{Damping Time:} \quad \tau_\zeta = \frac{\mathcal{E}_{\rm comp}}{W_{\rm comp}} = \frac{K\bar{n}\,{t_{\rm comp}}^2}{36\pi^2\,\zeta}}$$

Bulk visc is important if $\tau_\zeta \lesssim 10\,\text{ms}$

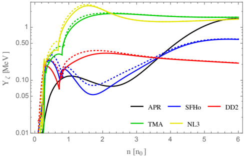
Is bulk visc big enough to matter?



There are high-amplitude density oscillations with $f \sim 1\,\mathrm{kHz}$ in regions at $T \sim 5$ to $10\,\mathrm{MeV}$



Suppose Direct Urca processes are suppressed at those temperatures and densities.
Then flavor equilibration via modified Urca will achieve its maximum value

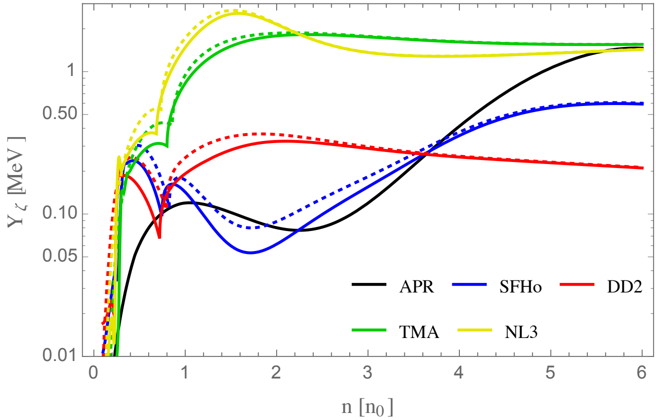


Max bulk visc from flavor equilibration is
$\zeta_{\mathrm{max}} = Y_\zeta \bar{n}\, t_{\mathrm{comp}}$

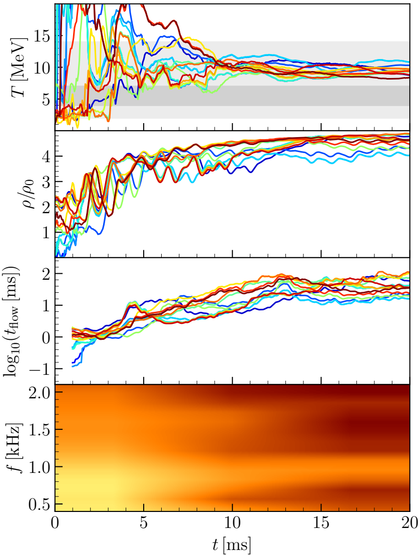
$$
\begin{aligned}
\tau_\zeta^{\mathrm{min}} &= \left(\frac{K}{36\pi^2\, Y_\zeta}\right) t_{\mathrm{comp}} \\
&\approx \boxed{3\ \ \mathrm{ms}} \left(\frac{t_{\mathrm{comp}}}{1\,\mathrm{ms}}\right) \left(\frac{K}{250\,\mathrm{MeV}}\right) \left(\frac{0.25\,\mathrm{MeV}}{Y_\zeta}\right)
\end{aligned}
$$

Max bulk visc “Y” factor

Typical value is in the 0.1 to 1 MeV range



State of post-merger matter



Temperature is in the range that maximizes bulk viscosity.
(assuming Modified Urca only)

Large amplitude $\sim 1\,kHz$ density oscillations during the first 5-10 ms

Density oscillation freq in kHz range

Summary

It is useful to have estimates of the equilibration times for various forms of dissipation, to decide which is the most important.

- Thermal equilibration: If neutrinos are trapped, and there are short-distance temp gradients then thermal transport might be fast enough to play a role.

$$\tau_\kappa^{(\nu)} \approx 700\text{ms} \left(\frac{z_{\rm typ}}{1\,\rm km}\right)^2 \left(\frac{T}{10\,\rm MeV}\right)^2 \left(\frac{0.1}{x_p}\right)^{1/3} \left(\frac{m_n^*}{0.8\,m_n}\right)^3 \left(\frac{\mu_e}{2\mu_\nu}\right)^2$$

- Shear viscosity: similar conclusion.
- Bulk viscosity: If Direct Urca processes remain suppressed at the relevant densities and temperatures, bulk viscosity will quickly damp density oscillations

$$\tau_\zeta^{\rm min} \approx 3\,\text{ms} \left(\frac{t_{\rm comp}}{1\,\rm ms}\right) \left(\frac{K}{250\,\text{MeV}}\right) \left(\frac{0.25\,\rm MeV}{Y_\zeta}\right)$$

The Future

- Incorporate bulk viscosity in numerical simulations
- What density and temperature range allows Direct Urca?
- Understand neutrino trapping. At what temp/density is there neutrino domination of thermal and shear viscous transport?
- Are there short-range gradients ($z_{\mathrm{typ}} \sim 0.1\,\mathrm{km}$) that would lead to rapid shear viscous or thermal equilibration?
- Explore the role of dissipation in the collapse of a single star to a denser “third family” or “twin star” configuration