Dense matter phases and new equilibria of compact stars

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Outline

Equilibria of compact objects

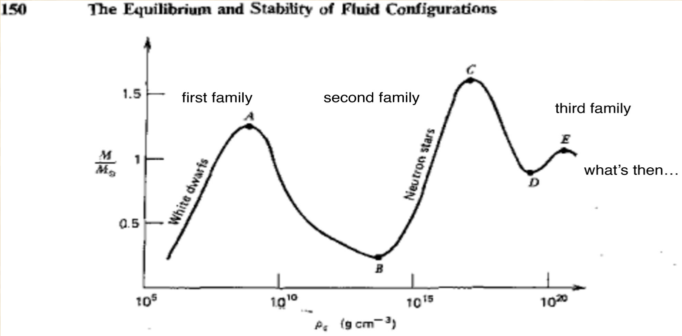


Figure 6.2 Schematic diagram showing the turning points in the mass versus central density diagram for equilibrium configurations of cold matter.

S. Shapiro, S. Teukolsky, "Black holes, White dwarfs and Neutron Stars"

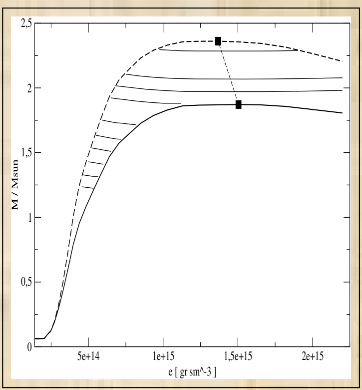
-*White dwarfs* -first family, $M \leq 1.5M_\odot$, [S. Chandrasekhar, L. Landau (1930-32)]
-*Neutron Stars* - second family, $M \leq 2M_\odot$, [Oppenhimer-Volkoff (1939)]
-*Hybrid Stars* - third family, $M \leq 2M_\odot$, [Gerlach (1968), Glendenning-Kettner (2000)]
- *Fourth Family?* $M \leq 2M_\odot$ if nature realizes NM→2SC→CFL [Alford, A.S. (2017)]

- Einstein's field equations:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -8\pi T_{\mu\nu},$$

- Energy-momentum tensor:

$$T_{\mu\nu} = -P(r) g_{\mu\nu} + [P(r) + \epsilon(r)]\, u_\mu u_\nu$$



TOV equations:

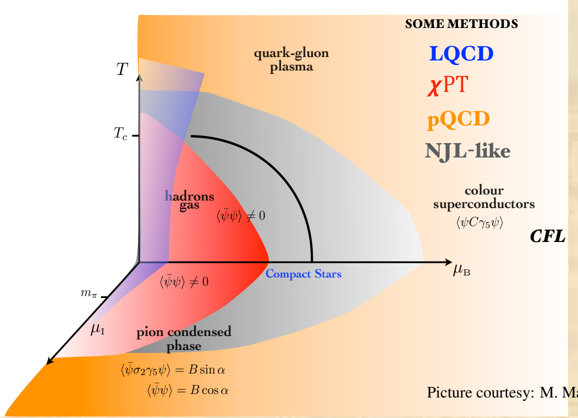
$$\frac{dP(r)}{dr} = -\frac{G\epsilon(r)M(r)}{c^2 r^2} \left(1 + \frac{P(r)}{\epsilon(r)}\right) \left(1 + \frac{4\pi r^3 P(r)}{M(r)c^2}\right) \left(1 - \frac{2GM(r)}{c^2 r}\right)^{-1}.$$

$$M(r) = 4\pi \int_0^r r^2 \epsilon(r) dr.$$

The Lagrangian of QCD is written for $\psi_q = (\psi_{qR}, \psi_{qG}, \psi_{qB})^T$ as

$$\mathcal{L}_{QCD} = \underbrace{\bar{\psi}^i_q (i\gamma^\mu)(D_\mu)_{ij}\psi^j_q - m_q \bar{\psi}^i_q \psi_{qi}}_{\text{quarks}} - \underbrace{\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}}_{\text{gluons(Yang-Mills)}} ,$$

where $\underbrace{(D_\mu)_{ij} = \delta_{ij}\partial_\mu - ig_s t^a_{ij} A^a_\mu}_{\text{covarinat derivative}}$, and $\underbrace{F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - 2q(A^\mu \times A^\nu)}_{\text{gluonic field (Yang-Mills) field tensor}}$



Picture courtesy: M. Mannarelli

Color-superconductivity within the NJL model

$$\begin{aligned}\mathcal{L}_{NJL} &= \underbrace{\bar\psi(i\gamma^\mu\partial_\mu - \hat m)\psi}_{\text{quarks}} + \underbrace{G_V(\bar\psi i\gamma^\mu\psi)^2}_{\text{vector}} + \underbrace{G_S\sum_{a=0}^{8}[(\bar\psi\lambda_a\psi)^2 + (\bar\psi i\gamma_5\lambda_a\psi)^2]}_{\text{scalar}-\text{pseudoscalar}}\\ &+ \underbrace{G_D\sum_{\gamma,c}[\bar\psi^a_\alpha i\gamma_5\epsilon^{\alpha\beta\gamma}\epsilon_{abc}(\psi_C)^b_\beta][(\bar\psi_C)^r_\rho i\gamma_5\epsilon^{\rho\sigma\gamma}\epsilon_{rsc}\psi^s_\sigma]}_{\text{pairing}}\\ &- \underbrace{K\left\{\det_f[\bar\psi(1+\gamma_5)\psi] + \det_f[\bar\psi(1-\gamma_5)\psi]\right\}}_{\text{t}'\text{Hooft interaction}},\end{aligned}$$

- quarks: ψ^a_α, color $a = r, g, b$, flavor ($\alpha = u, d, s$); mass matrix: $\hat m = \mathrm{diag}_f(m_u, m_d, m_s)$;
- other notations: λ_a, $a = 1, \ldots, 8$, $\psi_C = C\bar\psi^T$ and $\bar\psi_C = \psi^T C$, $C = i\gamma^2\gamma^0$.

Parameters of the model:

- G_S the scalar coupling and cut-off Λ are fixed from vacuum physics
- G_D is the di-quark coupling $\simeq 0.75 G_S$ (via Fierz) but free to change
- G_V and ρ_{tr} are treated as free parameters

QCD interactions pairing interactions and gaps

$$\Delta \propto \langle 0|\psi^a_{\alpha\sigma}\psi^b_{\beta\tau}|0\rangle$$

- Symmetric in space wave function (isotropic interaction)
- Antisymmetry in colors a, b for attraction
- Antisymmetry in spins σ, τ (Cooper pairs as spin-0 objects)
- Antisymmetry in flavors α, β

2SC phase:

Low densities, large m_s (strange quark decoupled)

$$\Delta(2SCs) \propto \Delta\epsilon^{ab3}\epsilon_{\alpha\beta} \qquad \delta\mu \ll \Delta,$$

Crystalline or gapless phases:

Intermediate densities, large m_s (strange quark decoupled)

$$\Delta(\text{cryst.}) \propto \epsilon_{\alpha\beta}\Delta_0\, e^{i\vec{Q}\cdot\vec{r}} \qquad \delta\mu \geq \Delta,$$

CFL phase:

High densities nearly massless u, d, s quarks

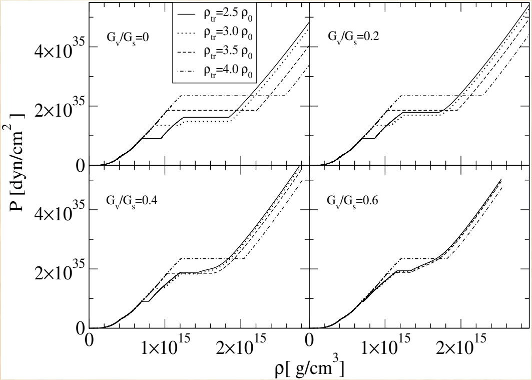
$$\Delta(CFL) \propto \langle 0|\psi^a_{\alpha L}\psi^b_{\beta L}|0\rangle = -\langle 0|\psi^a_{\alpha R}\psi^b_{\beta R}|0\rangle = \Delta\epsilon^{abC}\Delta\epsilon_{\alpha\beta C}.$$

EOS including (hyper)nuclear, 2SC and CFL phases of matter

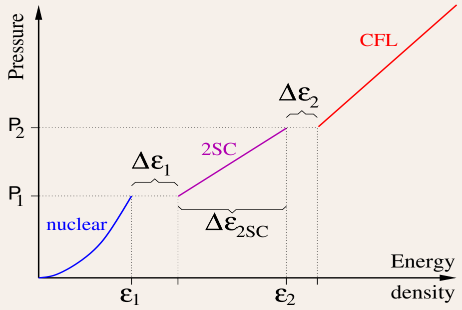
Choose Maxwell (large surface tension) or Glendenning (low surface tension) constructions. Matching condition for Maxwell is simply

$$P_N(\mu_B) = P_Q(\mu_B),$$

i.e., with low-density nuclear and high-density quark phases



Synthetic equations of state with constant speed of sound



- Instead of full NJL-model EoS with 2SC-CFL transition use synthetic EoS
- Realistic DD-ME2 EoS below the deconfinement (Colucci-Sedrakian EoS)
- Parametrize synthetic EoS via Constant Speed of Sound (CSS) parameterization (Alford-Han-Prakash 2013), also Haensel-Zdunik (2012).

Relativistic DFT theory

$$\mathcal{L} = \underbrace{\sum_B \bar{\psi}_B \left[\gamma^\mu \left(i\partial_\mu - g_{\omega BB}\omega_\mu - \frac{1}{2} g_{\rho BB} \boldsymbol{\tau} \cdot \boldsymbol{\rho}_\mu\right) - (m_B - g_{\sigma BB}\sigma)\right]\psi_B}_{\text{baryonic contribution}}$$
$$+ \underbrace{\frac{1}{2}\partial^\mu \sigma \partial_\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2}_{\text{scalar mesons}}$$
$$- \underbrace{\frac{1}{4}\omega^{\mu\nu}\omega_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu - \frac{1}{4}\boldsymbol{\rho}^{\mu\nu}\boldsymbol{\rho}_{\mu\nu} + \frac{1}{2} m_\rho^2 \boldsymbol{\rho}^\mu \cdot \boldsymbol{\rho}_\mu}_{\text{vector mesons}}$$
$$+ \underbrace{\sum_\lambda \bar{\psi}_\lambda (i\gamma^\mu \partial_\mu - m_\lambda)\psi_\lambda}_{\text{leptons}} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu},$$

- B-sum is over the baryonic octet $B \equiv p, n, \Lambda, \Sigma^{\pm,0}, \Xi^{-,0}$
- Meson fields include σ meson, $\boldsymbol{\rho}_\mu$-meson and ω_μ-meson
- Leptons include electrons, muons and neutrinos for $T \neq 0$

Fixing the couplings: nucleonic sector

$$g_{iN}(\rho_B) = g_{iN}(\rho_0)h_i(x), \qquad i=\sigma,\omega, \qquad h_i(x) = a_i\frac{1+b_i(x+d_i)^2}{1+c_i(x+d_i)^2}$$

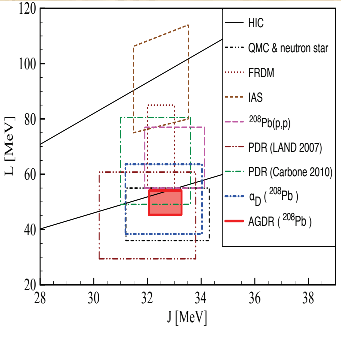
$$g_{\rho N}(\rho_B) = g_{\rho N}(\rho_0)\exp[-a_\rho(x-1)].$$

DD-ME2 parametrization of D. Vretenar, P. Ring et al. Phys. Rev. C 71, 024312 (2005).

	σ	ω	ρ
m_i [MeV]	550.1238	783.0000	763.0000
$g_{Ni}(\rho_0)$	10.5396	13.0189	3.6836
a_i	1.3881	1.3892	0.5647
b_i	1.0943	0.9240	—
c_i	1.7057	1.4620	—
d_i	0.4421	0.4775	—

Total number of parameters 8: boundary conditions on $h(x)$ at $x=1$.

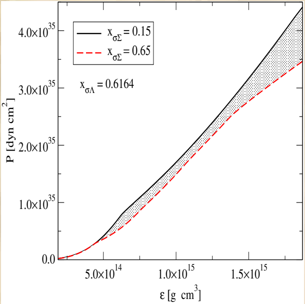
- saturation density $\rho_0 = 0.152\ \mathrm{fm}^{-3}$
- binding energy per nucleon $E/A = -16.14$ MeV,
- incompressibility $K_0 = 250.90$ MeV,
- symmetry energy $J = 32.30$ MeV,
- symmetry energy slope $L = 51.24$ MeV,
- symmetry incompressibility $K_{sym} = -87.19$ MeV

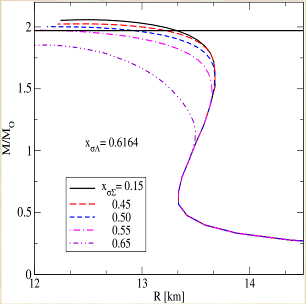


$$K_0 = k_F^2 \frac{\partial E/A}{\partial k^2}|_{k=k_F} = 9\rho_0^2 \frac{\partial^2 E/A}{\partial \rho^2}|_{\rho=\rho_0}, \qquad S(\rho) = \frac{1}{2}\frac{\partial^2 \epsilon/\rho}{\partial \delta^2}|_{\delta=0}.$$

$$S(\rho) = J + L\left(\frac{\rho-\rho_0}{3\rho_0}\right) + \frac{1}{2}K_{\rm sym}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2.$$

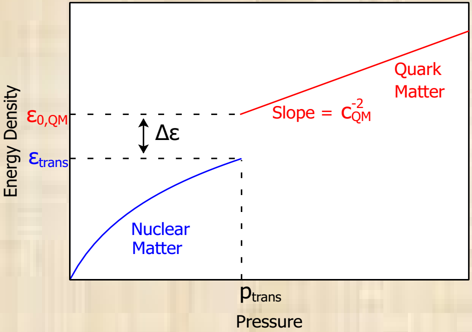
Equation of state and (hyper)nuclear stars





Zero temperature equations of state of hypernuclear matter for fixed $x_{\sigma\Lambda} = 0.6164$ and a range of values $0.15 \leq x_{\sigma\Sigma} \leq 0.65$. These values generate the shaded area, which is bound from below by the softest EoS (dashed red line) corresponding to $x_{\sigma\Sigma} = 0.65$ and from above by the hardest EoS (solid line) corresponding to $x_{\sigma\Sigma} = 0.15$.

CSS parameterization



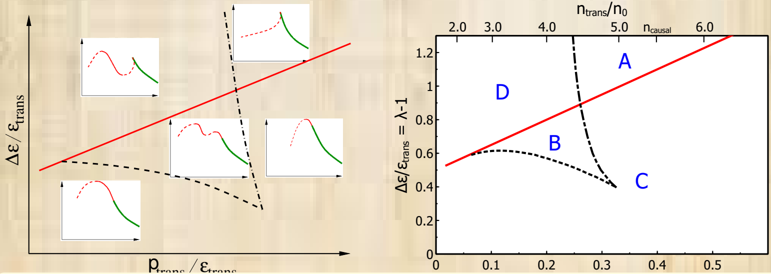
$$\varepsilon(p) = \begin{cases} \varepsilon_{\rm NM}(p) & p < p_{\rm trans} \\ \varepsilon_{\rm NM}(p_{\rm trans}) + \Delta\varepsilon + c_{\rm QM}^{-2}(p - p_{\rm trans}) & p > p_{\rm trans} \end{cases},$$

M. G. Alford, S. Han, M. Prakash, Phys. Rev. D 88, 083013 (2013).

J. Zdunik and P. Haensel A and A 551, A61, (2013).

Phase diagram in *M*-*R* space

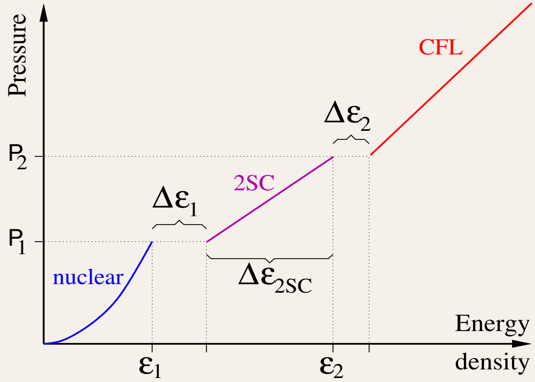
Phase diagram for hybrid star branches in the mass-radius relation of compact stars. The left panel shows schematically the possible topological forms of the mass-radius relation in each region of the diagram.



$$\frac{\Delta\varepsilon_{\rm crit}}{\varepsilon_{\rm trans}} = \frac{1}{2} + \frac{3}{2}\frac{p_{\rm trans}}{\varepsilon_{\rm trans}} .$$

M. G. Alford, S. Han, M. Prakash, Phys. Rev. D 88, 083013 (2013).

EoS with sequential phase transitions



Parameters of the models:

$$(\epsilon_1, P_1) \quad \Delta\varepsilon_1, \quad \Delta\varepsilon_{2SC} \quad (\varepsilon_2, P_2) \quad \Delta\varepsilon_2$$

Note that there are five independent parameters.

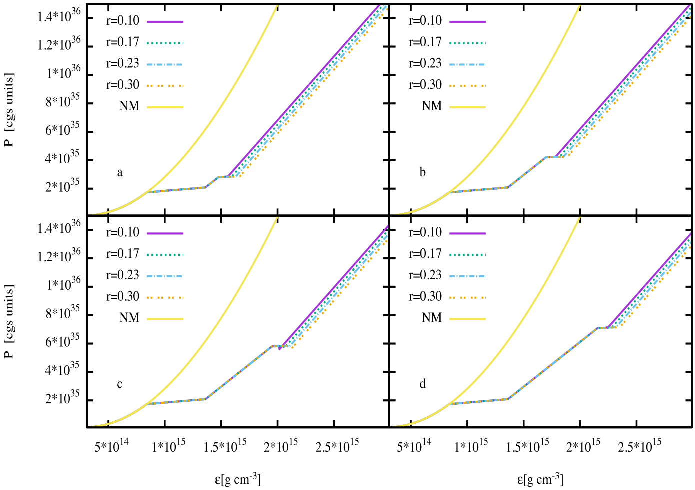
The EOS is analytically given

$$P(\varepsilon) = \begin{cases} P_1, & \varepsilon_1 < \varepsilon < \varepsilon_1 + \Delta\varepsilon_1 \\ P_1 + s_1\left[\varepsilon - (\varepsilon_1 + \Delta\varepsilon_1)\right], & \varepsilon_1 + \Delta\varepsilon_1 < \varepsilon < \varepsilon_2 \\ P_2, & \varepsilon_2 < \varepsilon < \varepsilon_2 + \Delta\varepsilon_2 \\ P_2 + s_2\left[\varepsilon - (\varepsilon_2 + \Delta\varepsilon_2)\right], & \varepsilon > \varepsilon_2 + \Delta\varepsilon_2\,. \end{cases}$$

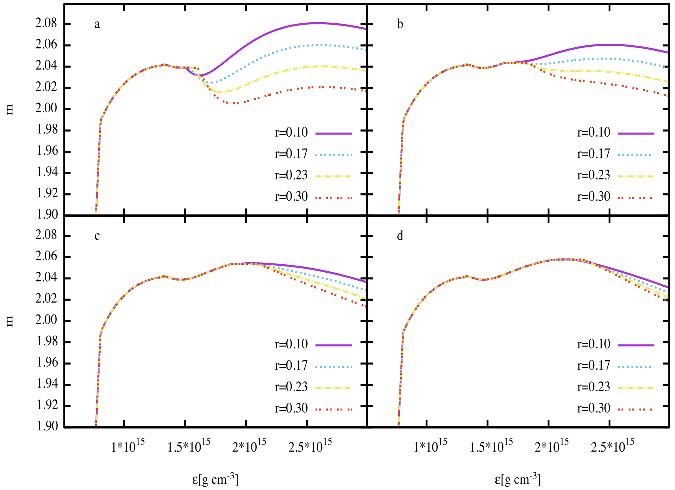
Need to specify:

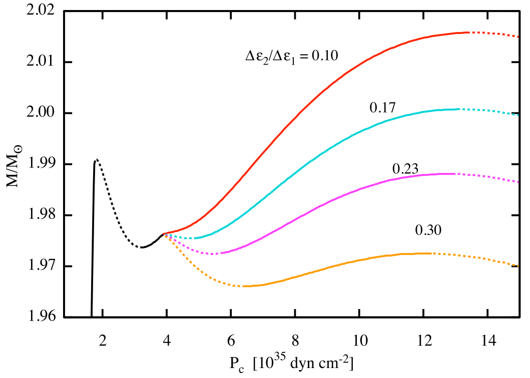
- the two speeds of sounds: s_1 and s_2
- the point of transition from NM to QM ε_1, P_1
- the magnitude of the first jump $\Delta\varepsilon_1$
- the size of the 2SC phase, i.e, the second transition point ε_2, P_2
- the size of the second jump $\Delta\varepsilon_2$

Varying parameters of EoS with sequential phase transition



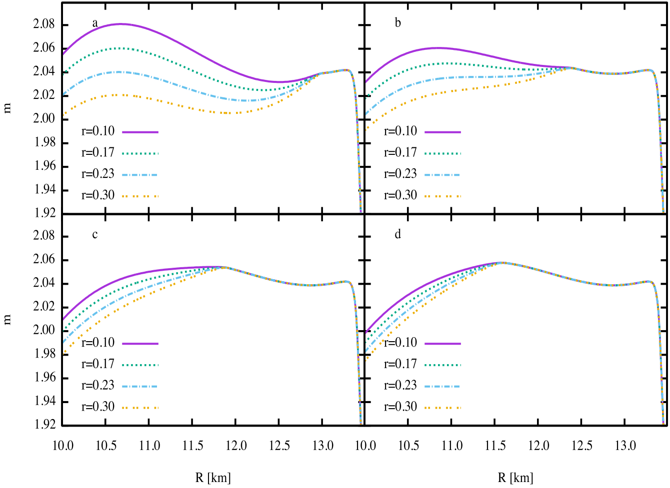
.... and resulting topologies of sequences

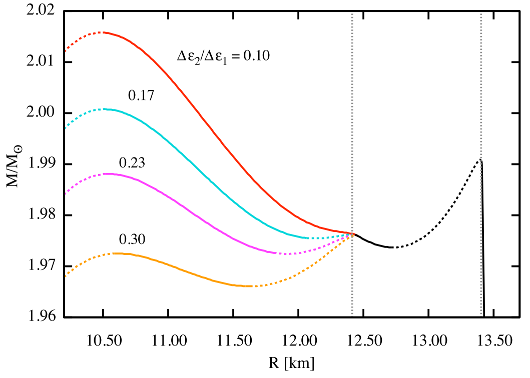




The stellar mass as a function of the star's central pressure for four different values of $\Delta\varepsilon_2$. The other parameters of the EOS are fixed at $P_1 = 1.7 \times 10^{35}$ dyn cm^{-2}, $s_1 = 0.7$, $\Delta\varepsilon_{2SC}/\varepsilon_1 = 0.27$, $\Delta\varepsilon_1/\varepsilon_1 = 0.6$, and $s_2 = 1$. The vertical dotted lines mark the two phase transitions at P_1 and P_2. Stable branches are solid lines, unstable branches are dashed lines. We see the emergence of separate 2SC and CFL hybrid branches along with the occurrence of triplets.

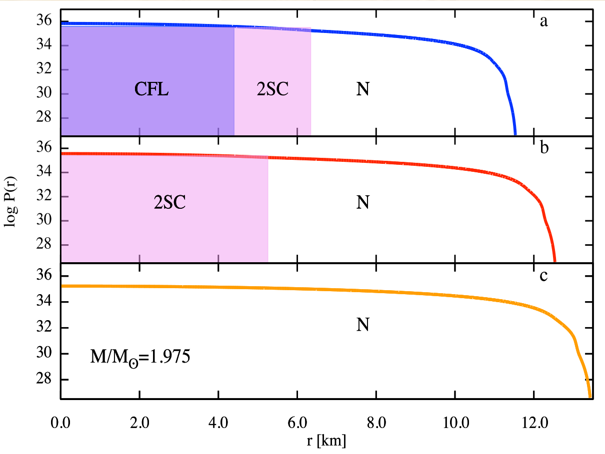
.... and resulting topologies of mass-radius relations





The *M*-*R* relations for the parameter values defined above . We have fixed the properties of the nuclear $\rightarrow$ 2SC transition and the speed of sound in 2SC and CFL matter. For the 2SC $\rightarrow$ CFL transition we have fixed the critical pressure and we vary the energy-density discontinuity $\Delta\varepsilon_2$. The separate 2SC and CFL hybrid branches are clearly visible, along with the occurrence of triplets.

Profiles of triplets stars (same mass)



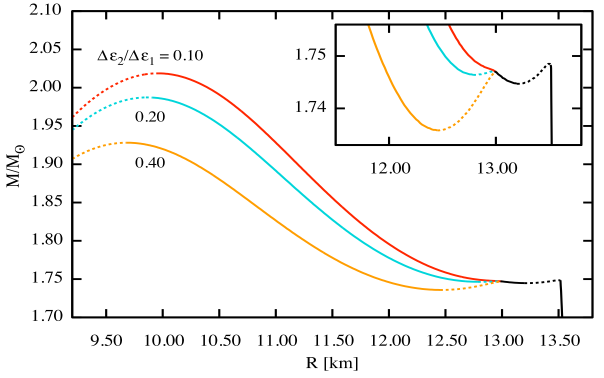
The profiles (here the log of pressure as a function of the internal radius) of the three members of a triplet with masses $M = 1.975\ M_\odot$. Here "N" means the nuclear phase. The parameter values are as above, with $\Delta\varepsilon_2/\Delta\varepsilon_1 = 0.23$.

Stability conditions for our models

$\Delta\varepsilon_2/\Delta\varepsilon_1$	$\Delta\varepsilon_1/\varepsilon_1$ 0.4	0.5	0.6	0.7
0.1	s, s	s, s	$\underbrace{us, s}_{\text{N-2SC}}$	$\underbrace{u, us}_{\text{N-CFL}}$
0.2	s, s	s, s	$\underbrace{us, us}_{\text{triplet}}$	$\underbrace{u, us}_{\text{N-CFL}}$
0.3	s, s	s, s	$\underbrace{us, us}_{\text{N-2SC;N-CFL}}$	$\underbrace{u, us}_{\text{N-CFL}}$
0.4	s, s	$\underbrace{s, us}_{\text{2SC-CFL}}$	$\underbrace{us, u}_{\text{N-2SC}}$	u, u
0.5	s, s	$\underbrace{s, us}_{\text{2SC-CFL}}$	$\underbrace{us, u}_{\text{N-2SC}}$	u, u

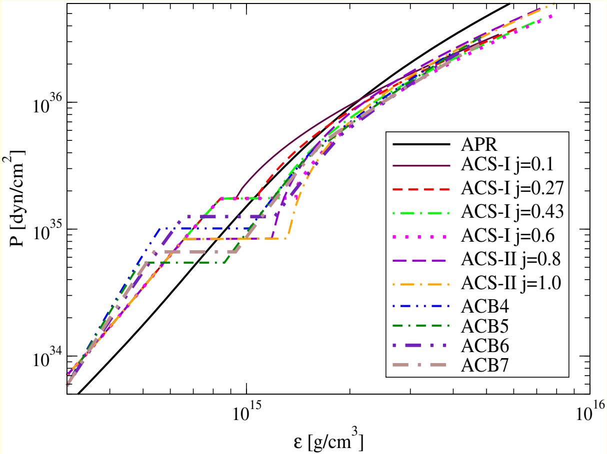
In each entry stable/unstable branches are referred by s/u, the 2SC and CFL phases are separated by comma, and the pressure increases from left to right. The presence of twin hybrid configurations or triplet configurations is marked by the underbraces with information about the involved phases ("N" means nuclear).

Lower mass triplets

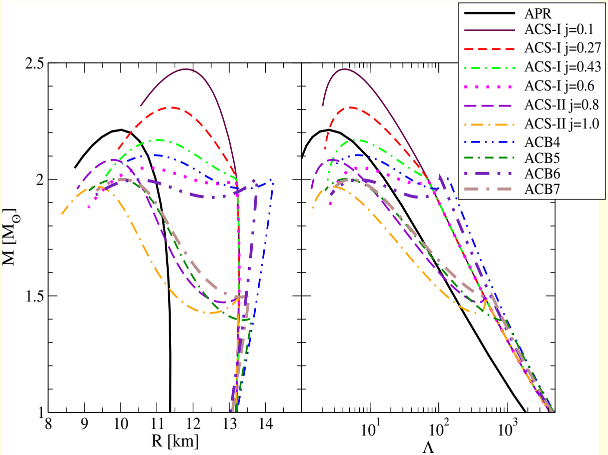


- Low-mass triplets via early transition NM$\rightarrow$ QM
- Still 2-solar mass members possible but only with the NM-2SC-CFL composition

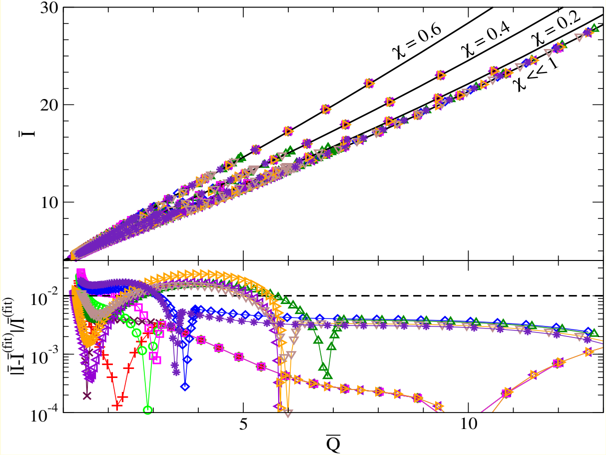
Low-mass hybrid stars and twins [V. Paschalidis, et al. Phys. Rev. D 97, 084038 (2018)]



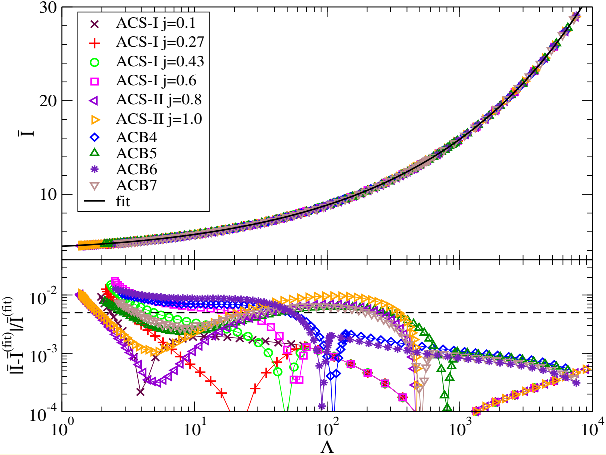
ACS - parametrizations based on CCS, ACB - piecewise polytropic EoS



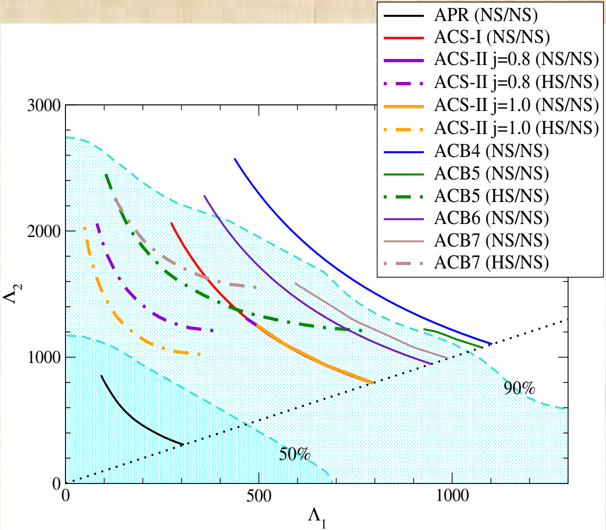
Mass-radius and deformability relations: Note low-mass twins at $1.4M_\odot$, which could be applicable to GW170917.



$I - Q$ relations for hybrid stars are to a high accuracy universal. Here $\chi = J/M^2$, with J the total angular momentum.

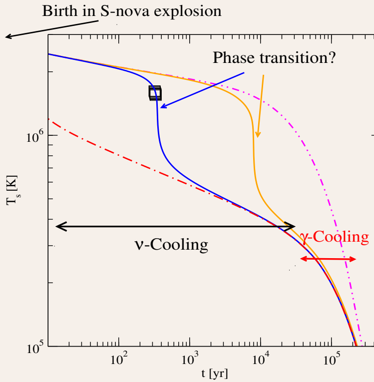


$I - \Lambda$ relations for hybrid stars are to a high accuracy universal.



Comparing with the LVC plot for the deformabilities.

Thermal history of Compact Stars (Cosmology in a single compact star)



$$\boxed{\frac{\partial S}{\partial t} + \vec{\nabla} \cdot \vec{q} = \mathcal{R} - \mathcal{L}_\nu - \mathcal{L}_\gamma} \qquad dS = c_V dT, \quad \vec{q} = \kappa \vec{\nabla} T$$

- Dissipative function $\mathcal{R} \propto \sigma(\nabla\phi)^2 + \kappa(\nabla T)^2/T + \ldots$
- Neutrino losses $\mathcal{L}_\nu$ from the bulk
- Photon losses $\mathcal{L}_\gamma$ from the surface

- Modified Urca/brems process

$$n + n \to n + p + e + \bar{\nu},$$
$$n + n \to n + p + \nu + \bar{\nu},$$

- Crustal bremsstrahlung

$$e + (A, Z) \to e + (A, Z) + \nu + \bar{\nu},$$

- Cooper pair-breaking-formation

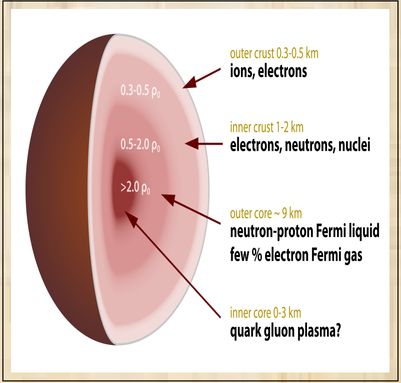
$$[NN] \to [NN] + \nu + \bar{\nu},$$

- Surface photo-emission

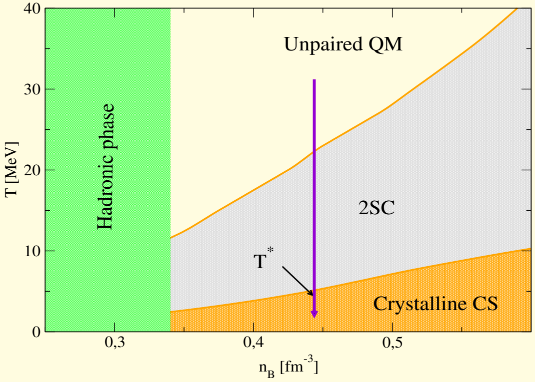
$$L_\gamma = 4\pi\sigma R^2 T^4$$

- Quark Urca process

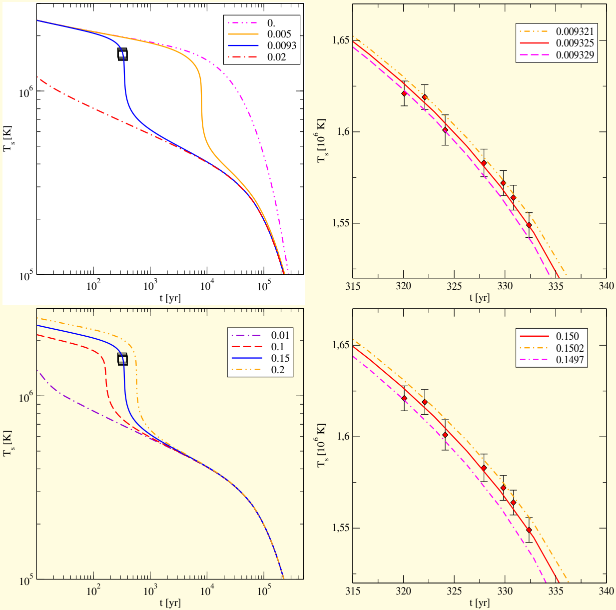
$$d \to u + e + \bar{\nu}, \quad u + e \to d + \nu.$$



Phase transition within the QCD phase diagram can take place from one pairing pattern to the other (e.g. 2SC to Crystalline)



- 2SC phase - fully gapped, no excitations, ν-emission strongly suppressed
- gapless (or crystalline) phase ν-emission enhanced



Fine tuned to the Cas A data using the phase transition temperature T^* andΔ_b.

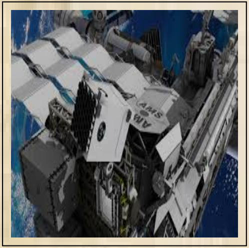
Conclusions

- Sequential phase transitions in the QCD phase diagram can lead to a new stable branch and a new family of compact stars, which is more compact than the ordinary hybrid stars.
- Triplets are possible: three distinct same mass stars configurations, differing in internal composition, radius, etc.
- Low-mass hybrid stars with $M \sim 1.4 M_{\odot}$ and twins can arise if the transition density to quark matter is low. This raises the possibility that GW170817 event involved a hybrid compact stars and that we can study dense QCD phases using GW signals from BNS.

Thanks to co-authors: Mark Alford, Vasileios Paschalidis, Kent Yagi, David Alvarez-Castillo, David B. Blaschke

Near future experimental advances:

- NICER (X-ray studies of neutron stars)
- LIGO-VIRGO (Gravitational waves from BNS and pulsars)
- SKA (radio timing of pulsars)



Theory questions:

- Dense QCD phases: static and dynamic properties
- Astrophysical properties of compact stars with quark phases
- Triplets and twins
- Gravity wave and QCD

Thank you for your attention!