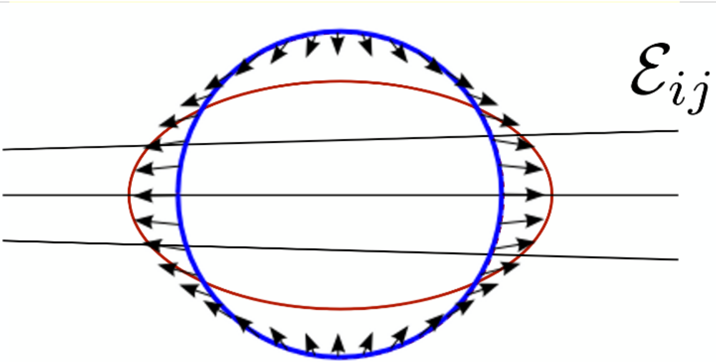
Tidal Deformability constraints of neutron stars and hybrid quark stars

Tianqi Zhao

Stony Brook University

CSQCD VII, 2018

Tidal field



$$Q_{ij} = -\Lambda \mathcal{E}_{ij}$$

Tidal deformability(Newtonian)

- Tidal field potential and quadruple moment in Newtonian physics are,

$$\Phi_{tidal} = \frac{1}{2}\varepsilon_{ij}x^i x^j \quad (1)$$

$$Q_{ij} = \int d^3x\delta\rho(x_i x_j - \frac{1}{3}\delta_{ij}) \quad (2)$$

- Quadruple moment is induced by tidal field linearly,

$$Q_{ij} = -\lambda\varepsilon_{ij} \quad (3)$$

where λ is tidal deformability with a unit of (mass)(length)^2/(time)^2.

- Two dimensionless parameters are defined from λ,

$$\Lambda = \lambda M^{-5} \quad \text{dimensionless tidal deformability} \quad (4)$$

$$k_2 = \frac{3}{2}\lambda R^{-5} \quad \text{tidal Love number} \quad (5)$$

where $c = G = 1$.

Tidal deformability(GR generized)

- Total gravitational potential in Newtonian physics are,

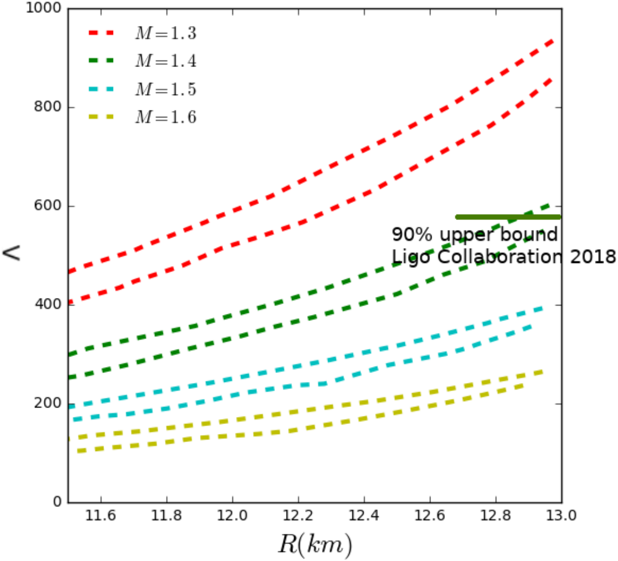
$$\Phi = \frac{1}{2}\varepsilon_{ij}x^i x^j - \frac{M}{r} - \frac{3}{2}\frac{Q_{ij}x^i x^j}{r^5} \tag{6}$$

- We know general relativity reduce to Newtonian gravity at weak(far) field. And tt component of metric plays the exact role of Newtonian potential. Thus is should be expanded in powers of r to match Newtonian definition of quadruple moment and tidal deformability.

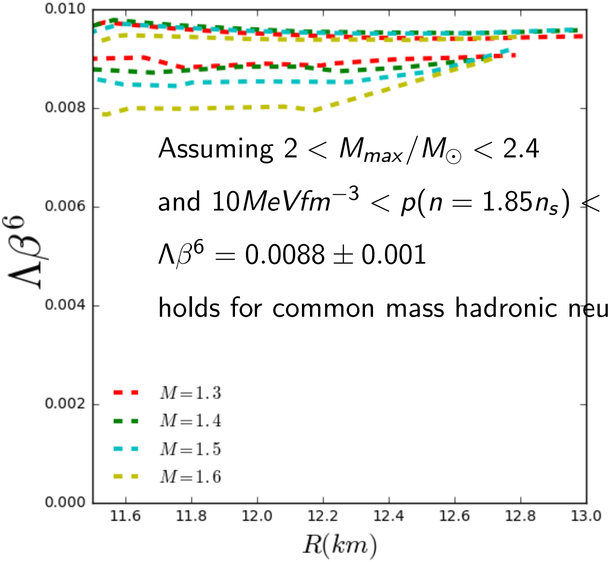
$$-\frac{1+g_{00}}{2} = \frac{1}{2}\varepsilon_{ij}x^i x^j - \frac{M}{r} - \frac{3}{2}\frac{Q_{ij}x^i x^j}{r^5} + \sum C_n r^n \quad (n \neq 2, -1, -3) \tag{7}$$

- g_{00} can be solved by introducing a linear $Y_{l=2}^{m=0}(\theta, \phi)$ perturbation on spherical symmetric metric(TOV metric). And the ratio between its (n=2) order and (n=-3) order coefficient defines tidal deformability.

Tidal deformability of hadronic NS



Tidal deformability of hadronic NS



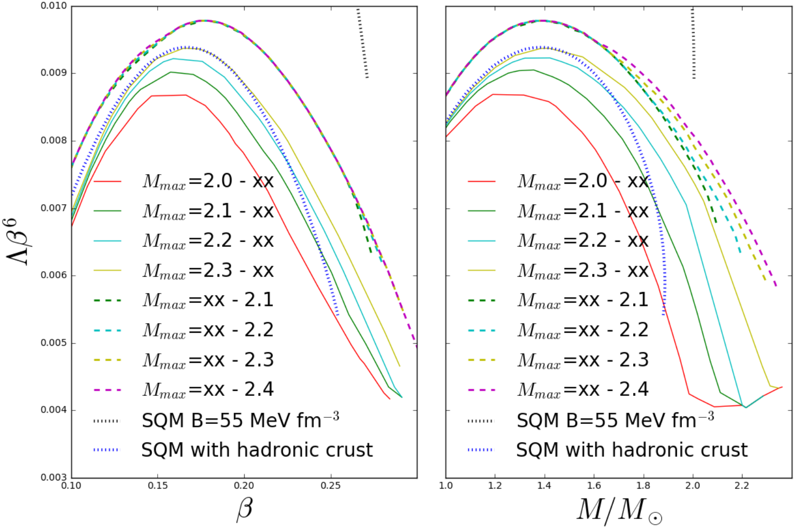
Assuming $2 < M_{max}/M_{\odot} < 2.4$

and $10MeVfm^{-3} < p(n = 1.85n_s) < 20MeVfm^{-3}$

$\Lambda\beta^6 = 0.0088 \pm 0.001$

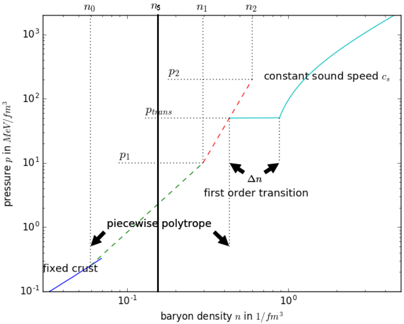
holds for common mass hadronic neutron star

Tidal deformability of hadronic NS



Hybrid star with first order phase transition

- BPS as fixed crust EoS up to $n_0 \approx n_s/2.7$
- Three piecewise polytropic EoS devided by $n_1 = 1.85n_s, n_2 = 3.74n_s$ (J.S. Read 2008)
- Constant sound speed(CSS) is used for quark core.
- A first order transition is assumed to happened between n_s and $3.74n_s$, or $p_{trans} < 250MeV/fm^3$. Chemical equilibrium is assumed at boundary,

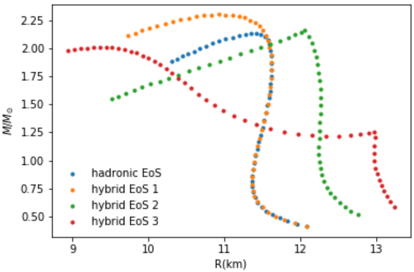


$$p_{hardron} = p_{quark}, \ \mu_{hardron} = \mu_{quark} \tag{8}$$

$$\epsilon(p) = \begin{cases} \epsilon_{poly}(p) & \text{if } p < p_{trans} \\ \epsilon_{poly}(p_{trans}) + \Delta\epsilon + \frac{p - p_{trans}}{c_s^2} & \text{if } p < p_{trans} \end{cases} \tag{9}$$

Hybrid star with first order phase transition

- $p_1 = p(n_1)$ is closely related with radius of a typical neutron star, and is constrained by neutron matter calculation.
- $p_2 = p(n_2)$ define stiffness below transition, bounded by causality at transition.
- Sound speed c_s^2 affect maximum mass of hybrid star.
- Energy discontinuity $\Delta\varepsilon > 0$(stability), is bounded by requiring $M_{max} > 2.01 M_{\odot}$.



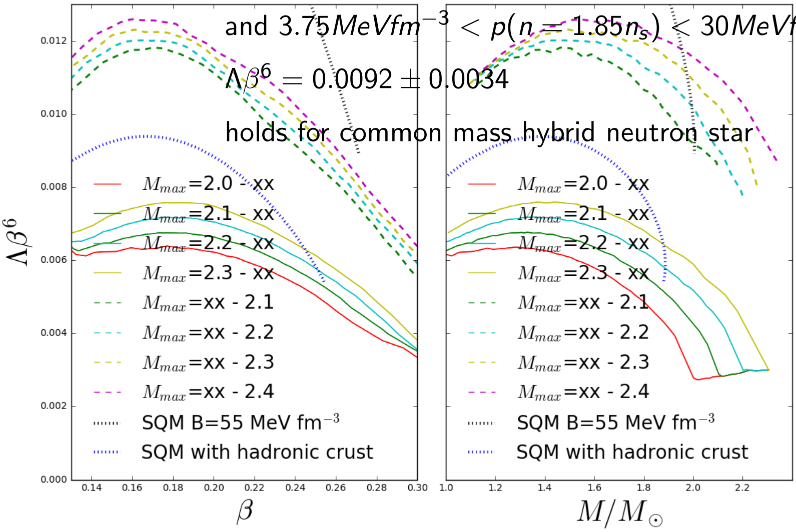
Tidal Deformability of Hybrid NS

Assuming $2 < M_{max}/M_\odot < 2.4$, $n_{trans} > n_s$

and $3.75 MeV fm^{-3} < p(n = 1.85 n_s) < 30 MeV fm^{-3}$,

$\Lambda\beta^6 = 0.0092 \pm 0.0034$

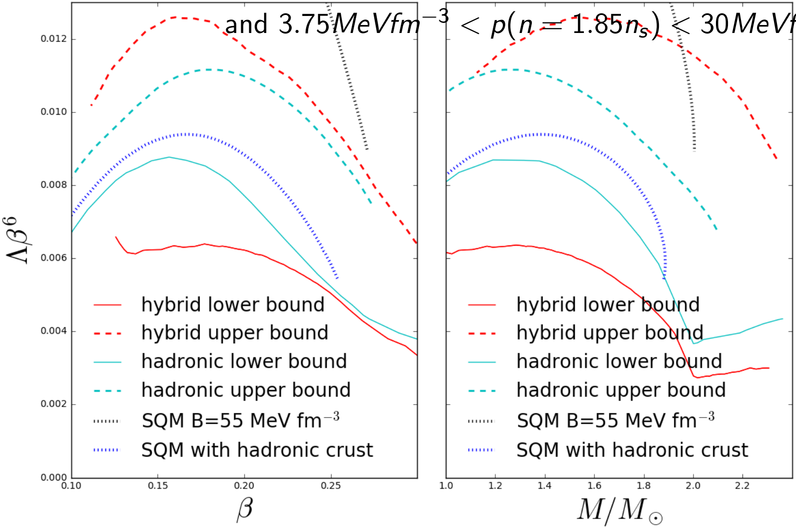
holds for common mass hybrid neutron star



Tidal Deformability of Hybrid NS

Assuming $2 < M_{max}/M_{\odot} < 2.4$, $n_{trans} > n_s$

and $3.75 MeVfm^{-3} < p(n = 1.85 n_s) < 30 MeVfm^{-3}$



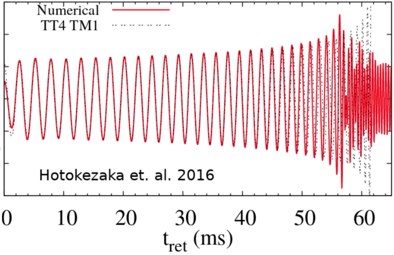
Tidal deformability in binary merger GW waveform

- Oscillating Quadruple moments of neutron star due to excitation of periodic tidal fields contributes to phase shift in GW form.
- Quadruple oscillating contribute to GW radiation reaction,

$$\dot{E}(\omega) = -\frac{1}{5} < \dddot{Q}^T_{ij} \dddot{Q}^T_{ij} >= -\frac{32}{5} M^{4/3} \mu^2 \omega^{10/3} [1 + g(\omega)] \qquad (10)$$

- By evaluating stable orbit, contribution of quadruple oscillating to total energy of the binary can be calculated,

$$E(\omega) = -M^{1/3} \omega^{-2/3} [1 + f(\omega)] \qquad (11)$$



- Using formula $\frac{d^2\Phi}{d\omega^2} = 2(\frac{dE}{d\omega})/\dot{E}$, tidal phase correction can be derived,

$$\delta\Phi = -\frac{9}{16} \frac{\omega^{5/3}}{\mu M^{7/3}} [(\frac{12 m_2 + m_1}{m_1} \lambda_1 + \frac{12 m_1 + m_2}{m_2} \lambda_2] \qquad (12)$$

Flanagan+ 2008

Binary tidal deformability

- At leading order, phase shift of GW is proportional to the binary tidal deformability,

$$\bar{\Lambda} = \frac{16}{13} \frac{(12q+1)\Lambda_1 + (12+q)q^4\Lambda_2}{(1+q)^5} \tag{13}$$

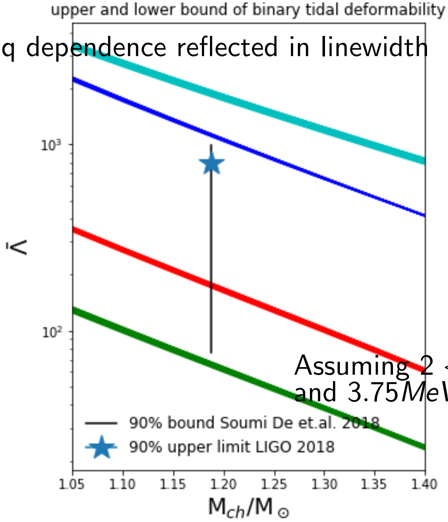
  where $q = m_2/m_1 < 1$.
- Statistics are required in order to make any conclusion for binary tidal deformability. Thus, prior distribution of $\tilde{\Lambda}, \Lambda_1, \Lambda_2$ is important.
- We set up bounds of $\tilde{\Lambda}$ and Λ_2/Λ_1 as a function of chirp mass M_{ch} and q, in the scenario of hadronic star and hybrid star respectively.

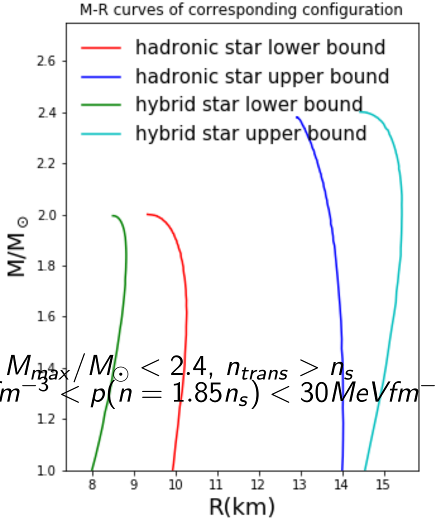
$$\tilde{\Lambda}_{lower}(M_{ch}, q) < \tilde{\Lambda} < \tilde{\Lambda}_{upper}(M_{ch}, q) \tag{14}$$

$$(\Lambda_2/\Lambda_1)_{lower}(M_{ch}, q) < (\Lambda_2/\Lambda_1) < (\Lambda_2/\Lambda_1)_{upper}(M_{ch}, q) \tag{15}$$

Bounds of binary tidal deformability $\tilde{\Lambda}$

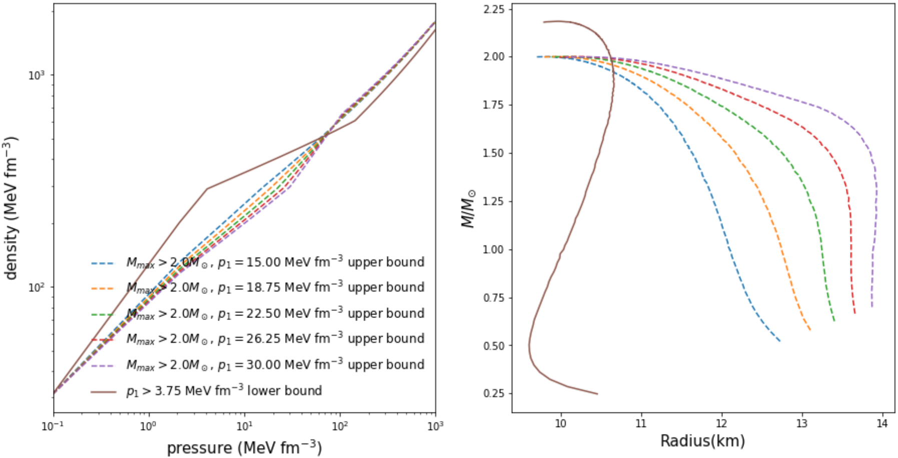
q dependence reflected in linewidth



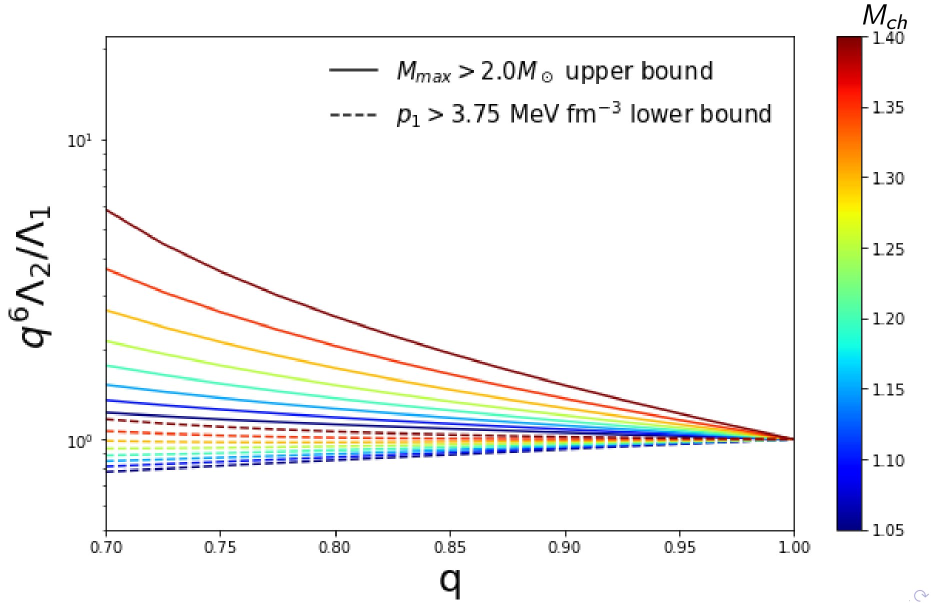


Assuming $2 < M_{max}/M_{\odot} < 2.4$, $n_{trans} > n_s$ and $3.75 MeVfm^{-3} < p(n = 1.85n_s) < 30 MeVfm^{-3}$,

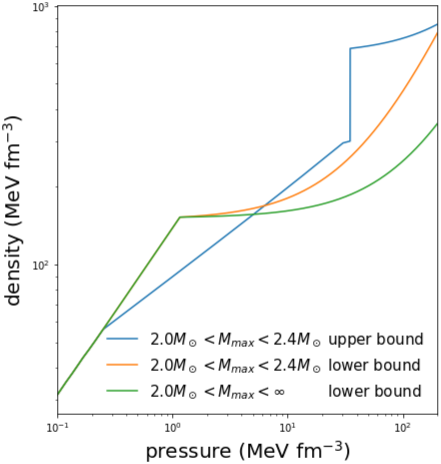
EoS and M-R curves for hadronic bounds of Λ_2/Λ_1

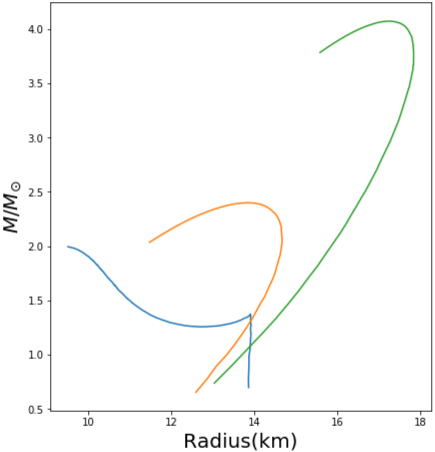


Hadronic bounds of binary tidal deformability ratio $\tilde{\lambda}_2/\tilde{\lambda}_1$

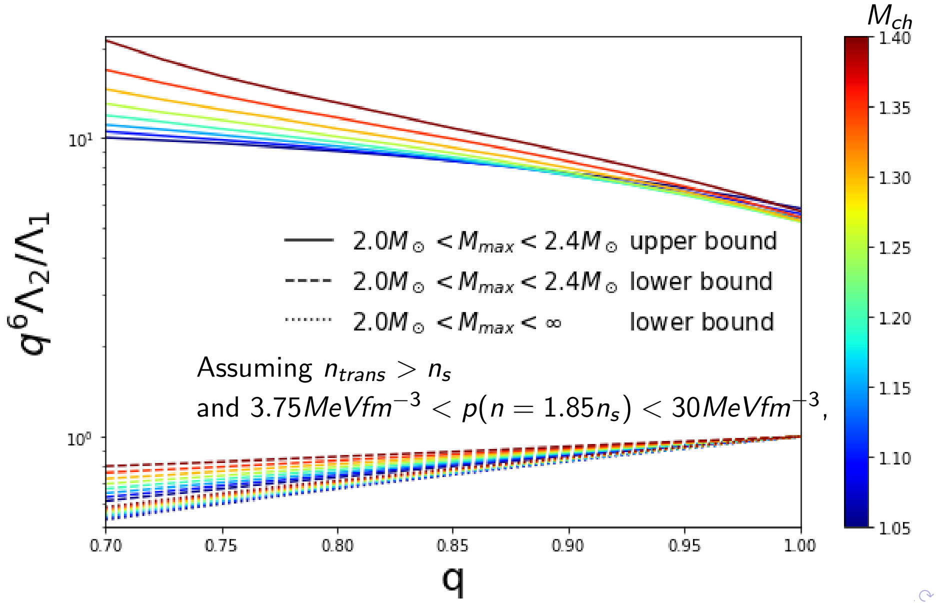


Hybrid star bounds of binary tidal deformability ratio $\tilde{\lambda}_2/\tilde{\lambda}_1$

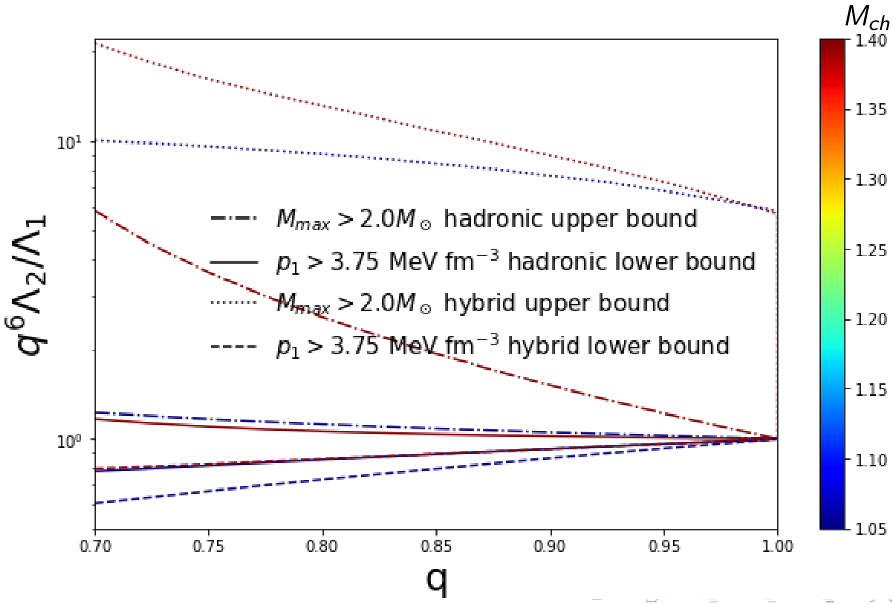




Hybrid star bounds of binary tidal deformability ratio $\tilde{\lambda}_2/\tilde{\lambda}_1$



Bounds of binary tidal deformability ratio $\tilde{\lambda}_2/\tilde{\lambda}_1$



Summary

- Tidal deformability is a measure of compactness.
- Tidal deformability-compactness relation is broaden when hybrid neutron star was taken into account.
- Binary tidal deformability is a 'average' tidal deformability of the two star with more weight on the massive one. It appear as perturbation of phase shift in GW observation.
- Together with mass knowledge, tidal deformability will provide us radius of neutron star, eventually the information of EoS around $n = 1 - 2n_s$