Vector–interaction–enhanced bag model.

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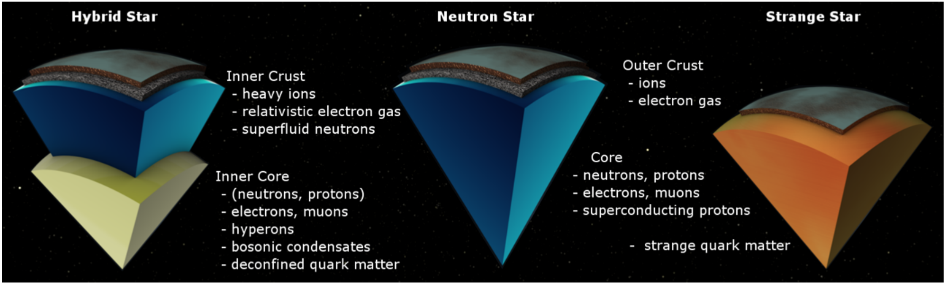
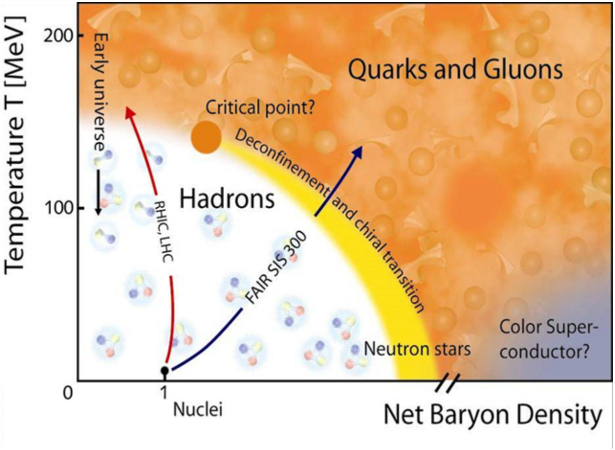
Overview

Motivation



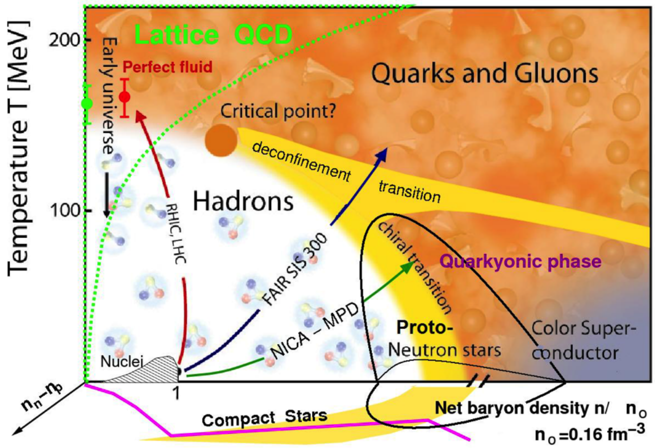
[1] Image courtesy of Thomas Klähn

QCD phase diagram[2]



[2] Image retrieved from `http://www.gsi.de`

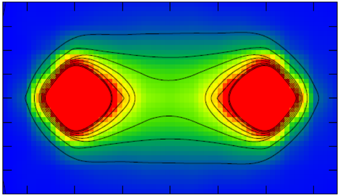
QCD phase diagram[3]



[3] Image retrieved from http://theor0.jinr.ru/twiki-cgi/view/NICA.

Solutions - Exact

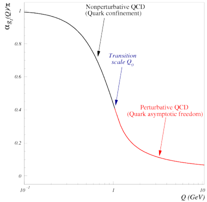
Lattice QCD[4]



Problems:

- Fermion doubling
- Numerical sign problem

Perturbative QCD[5]



Problems:

- Only accurate for very high energies
- Not applicable to phase transitions

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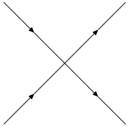
[4] Image retrieved from `https://arxiv.org/pdf/0912.3181.pdf`
[5] Image retrieved from `https://arxiv.org/pdf/1509.03112.pdf`

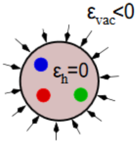
Solutions - Effective models

Nambu–Jona-Lasino model



- Assumes only contact interactions between quarks
- Exhibits $D\chi SB$ but no confinement

Bag model[6]



- Designed to mimic confinement
- Assumes constant quark masses

Both models are inspired by, but not originate from QCD!

[6] Image retrieved from `https://arxiv.org/pdf/0811.2024.pdf`

Dyson–Schwinger equations

The basic concept: $\int_a^b \frac{d}{dz} f(z) dz = 0$

This can be applied to the QCD generating functional

$$Z = \int [D\Phi] e^{iS + i \int d^4x (J_a^\mu G_\mu^a + \bar{\eta}\psi + \eta\bar{\psi})}$$

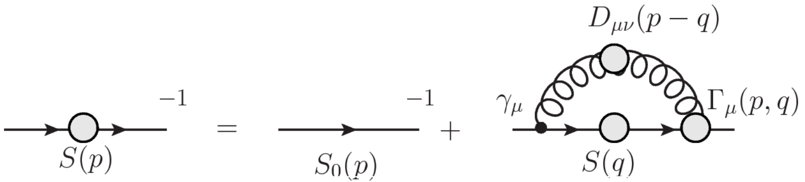
giving us

$$\frac{dZ}{d\eta} = 0$$

which is helpful because

$$G^{(N)}(x_1, ..., x_N) = \frac{(-i)^N}{Z[0]} \frac{\partial^N Z[J]}{\partial J(x_1) ... \partial J(x_N)}\bigg|_{J=0}$$

The Quark Dyson–Schwinger equation



One particle propagator in–medium

$$S^{-1}(p,\mu) = i\vec{\gamma}\vec{p} + i\gamma_4(p_4 + i\mu) + m + \Sigma(p,\mu)$$

Self–energy term

$$\Sigma(p,\mu) = \int \frac{d^4q}{(2\pi)^4} g^2 D_{\rho\sigma}(p-q)\gamma^\rho \frac{\lambda^\alpha}{2} S(q)\Gamma^\sigma_\alpha(p,q)$$

General form of the propagator:

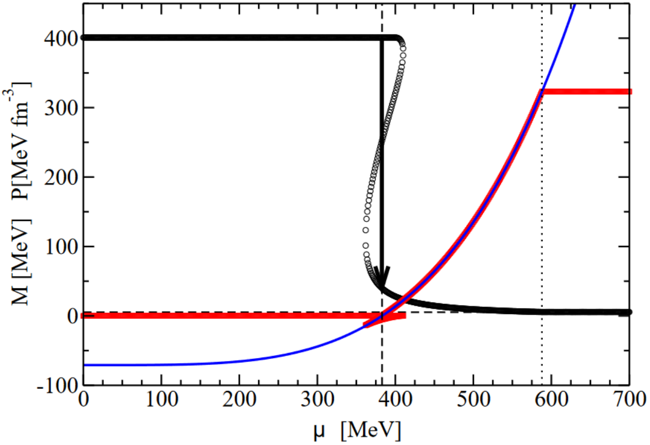
$$S^{-1}(p,\mu) = i\bar{\gamma}\bar{p}A(p,\mu) + i\gamma_4\tilde{p}_4 C(p,\mu) + B(p,\mu)$$

Truncation

$$g^2 D_{\rho\sigma}(p-q) = \delta_{\rho\sigma}\frac{1}{m_G^2}\Theta(\Lambda^2 - \vec{p}^2)$$

DSE results:

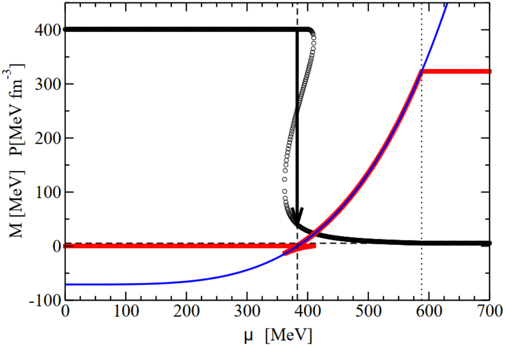
$$\begin{cases} A(p,\mu) = 1 \\ B(p,\mu) = m + \frac{16N_c}{9m_G^2}\int_\Lambda \frac{d^4q}{(2\pi)^4}\frac{B(q,\mu)}{\vec{q}^2A^2(q,\mu)+\tilde{q}_4^2C^2(q,\mu)+B^2(q,\mu)} \\ \tilde{p}_4^2 C(p,\mu) = \tilde{p}_4 + \frac{8N_c}{9m_G^2}\int_\Lambda \frac{d^4q}{(2\pi)^4}\frac{\tilde{p}_4\tilde{q}_4 C(q,\mu)}{\vec{q}^2A^2(q,\mu)+\tilde{q}_4^2C^2(q,\mu)+B^2(q,\mu)} \end{cases}$$



[7] Klähn, Fischer, Astrophys.J. 810 (2015) 2, 134

vBag

The chiral bag[8]



vBag EoS

$$\mu_f = \mu_f^* + K_v n_{FG,f}(\mu_f^*)$$

$$P_f(\mu_f) = P_{FG,f}(\mu_f^*) + \frac{K_v}{2} n_{FG,f}^2(\mu_f^*) - B_{\chi,f}$$

$$P^Q = \sum P_f(\mu_f)$$

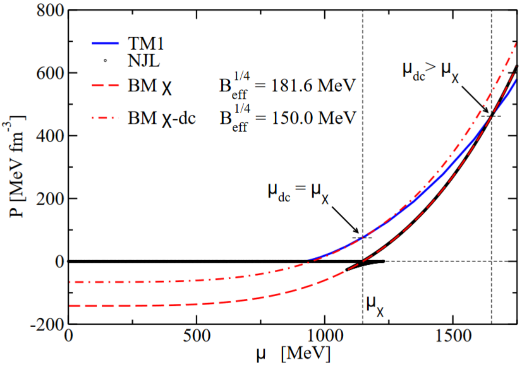
$$\epsilon_f(\mu_f) = \epsilon_{FG,f}(\mu_f^*) + \frac{K_v}{2} n_{FG,f}^2(\mu_f^*) + B_{\chi,f}$$

$$\epsilon^Q = \sum \epsilon_f(\mu_f)$$

$$n_{v,f}(\mu_f) = n_{FG,f}(\mu_f^*)$$

[8] Klähn, Fischer, Astrophys.J. 810 (2015) 2, 134

The (de)confinement bag[9]



vBag EoS

$$\mu_f = \mu_f^* + K_v n_{FG,f}(\mu_f^*)$$

$$P_f(\mu_f) = P_{FG,f}(\mu_f^*) + \frac{K_v}{2} n_{FG,f}^2(\mu_f^*) - B_{\chi,f}$$

$$P^Q = \sum P_f(\mu_f) + B_{dc}$$

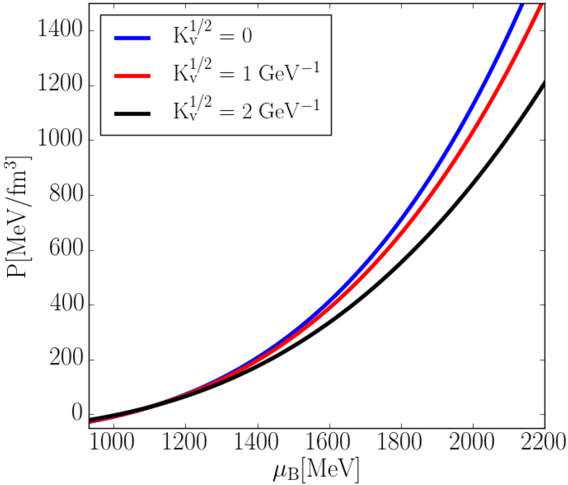
$$\epsilon_f(\mu_f) = \epsilon_{FG,f}(\mu_f^*) + \frac{K_v}{2} n_{FG,f}^2(\mu_f^*) + B_{\chi,f}$$

$$\epsilon^Q = \sum \epsilon_f(\mu_f) - B_{dc}$$

$$n_{v,f}(\mu_f) = n_{FG,f}(\mu_f^*)$$

[9] Klähn, Fischer, Astrophys.J. 810 (2015) 2, 134

Vector repulsion[10]



vBag EoS

$$\mu_f = \mu_f^* + K_v n_{FG,f}(\mu_f^*)$$

$$P_f(\mu_f) = P_{FG,f}(\mu_f^*) + \frac{K_v}{2} n_{FG,f}^2(\mu_f^*) - B_{\chi,f}$$

$$P^Q = \sum P_f(\mu_f) + B_{dc}$$

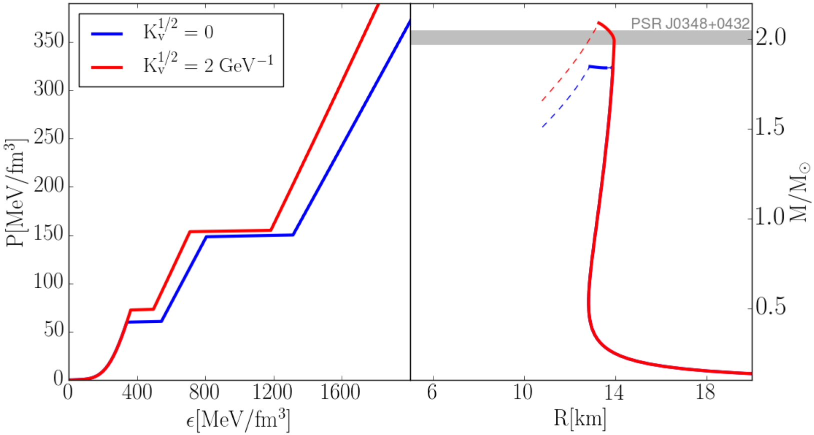
$$\epsilon_f(\mu_f) = \epsilon_{FG,f}(\mu_f^*) + \frac{K_v}{2} n_{FG,f}^2(\mu_f^*) + B_{\chi,f}$$

$$\epsilon^Q = \sum \epsilon_f(\mu_f) - B_{dc}$$

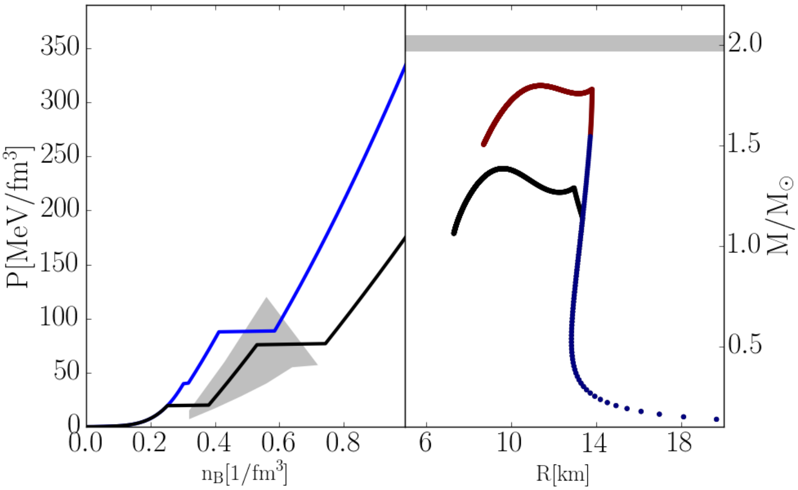
$$n_{v,f}(\mu_f) = n_{FG,f}(\mu_f^*)$$

[10] Cierniak, Klähn, Fischer, Bastian, Universe 4 (2018) 2, 30

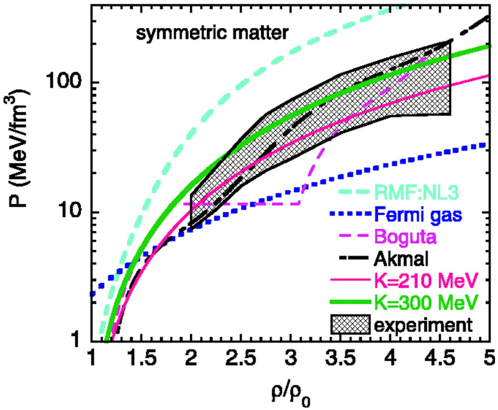
Mass–radius relation[11]



[11] Cierniak, Klähn, Fischer, Bastian, Universe 4 (2018) 2, 30

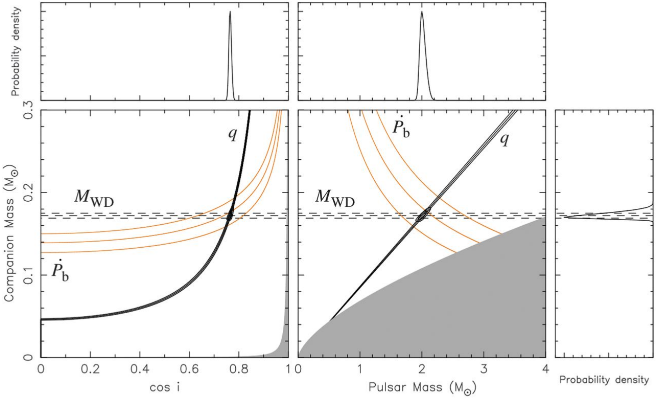

350
300
250
200
150
100
50
0
P[MeV/fm³]
0.0
0.2
0.4
0.6
0.8
n_B[1/fm³]
6
10
14
18
R[km]
2.0
1.5
1.0
0.5
M/M⊙

Danielewicz constraint[12]



[12] Danielewicz, Lacey, Lynch, Science 298 (2002)

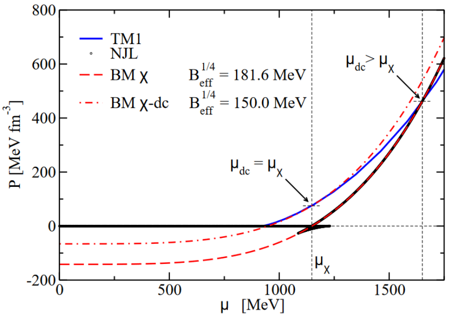
Antoniadis pulsar[13]



[13] Antoniadis, et al., Science 340 (2013)

Finite temperature

vBag at $T \neq 0$[14,15]



vBag EoS

$$\mu_f = \mu_f^* + K_v n_{FG,f}(\mu^*)$$

$$P_f(T,\mu_f) = P_{FG,f}(T,\mu_f^*) + \frac{K_v}{2} n_{FG,f}^2(\mu_f^*) - B_{\chi,f}$$

$$P^Q = \sum P_f(T,\mu_f) + B_{dc}(T)$$

$$\epsilon_f(T,\mu_f) = \epsilon_{FG,f}(T,\mu_f^*) + \frac{K_v}{2} n_{FG,f}^2(\mu_f^*) + B_{\chi,f}$$

$$\epsilon^Q = \sum \epsilon_f(T,\mu_f^*) - B_{dc}(T) + T\frac{\partial B_{dc}(T)}{\partial T}$$

$$n_f(\mu_f) = n_{FG,f}(\mu_f^*)$$

$$s_f(T,\mu_f) = \left.\frac{\partial P_f(T,\mu_f)}{\partial T}\right|_{\mu_f}$$

$$s(T,\mu_f) = \sum s_f(T,\mu_f) + \frac{\partial B_{dc}(T)}{\partial T}$$

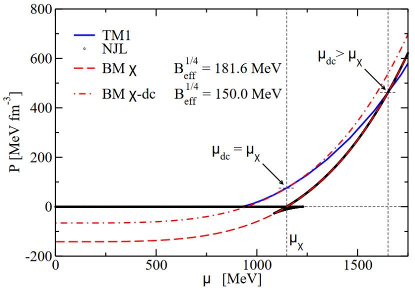
$$\mu_B = \mu_u + 2\mu_d$$

$$n_B = \frac{\partial P}{\partial \mu_B}$$

[14] Klähn, Fischer, Astrophys.J. 810 (2015) 2, 134

[15] Fischer, Klähn, Hempel, Eur.Phys.J. A52 (2016) 8, 225

vBag at $T \neq 0$ and $\mu_C \neq 0$[16]



vBag EoS

$$\mu_f = \mu_f^* + K_v n_{FG,f}(\mu^*)$$

$$P_f(T,\mu_f) = P_{FG,f}(T,\mu_f^*) + \frac{K_v}{2} n_{FG,f}^2(\mu_f^*) - B_{\chi,f}$$

$$P^Q = \sum P_f(T,\mu_f) + B_{dc}(T)$$

$$\epsilon_f(T,\mu_f) = \epsilon_{FG,f}(T,\mu_f^*) + \frac{K_v}{2} n_{FG,f}^2(\mu_f^*) + B_{\chi,f}$$

$$\epsilon^Q = \sum \epsilon_f(T,\mu_f^*) - B_{dc}(T) + T\frac{\partial B_{dc}(T)}{\partial T} + \mu_C \frac{\partial B_{dc}(T,\mu_C)}{\partial \mu_c}$$

$$n_f(\mu_f) = n_{FG,f}(\mu_f^*)$$

$$s_f(T,\mu_f) = \left.\frac{\partial P_f(T,\mu_f)}{\partial T}\right|_{\mu_f}$$

$$s(T,\mu_f) = \sum s_f(T,\mu_f) + \frac{\partial B_{dc}(T)}{\partial T}$$

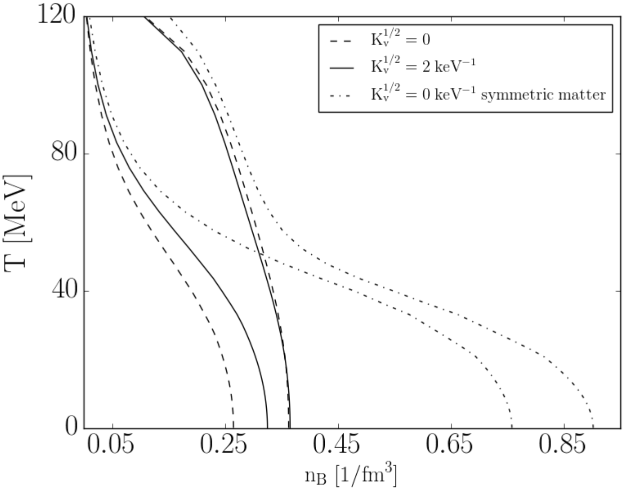
$$\mu_B = \mu_u + 2\mu_d$$

$$n_B = \frac{\partial P}{\partial \mu_B}$$

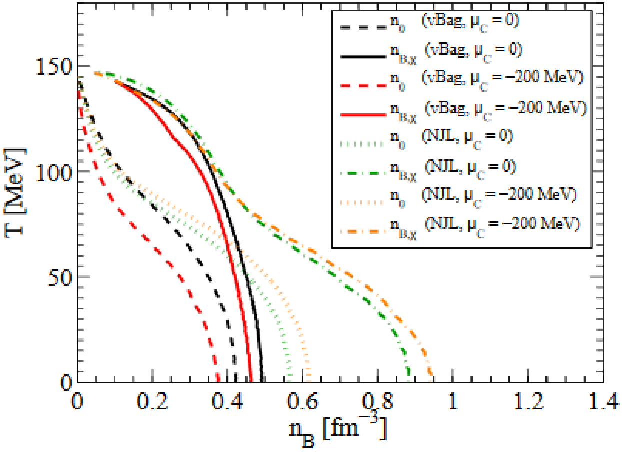
$$\mu_c = \mu_u - \mu_d$$

[16] Klähn, Fischer, Hempel, Astrophys.J. 836 (2017) 1, 89

Phase diagram



Phase diagram[17]



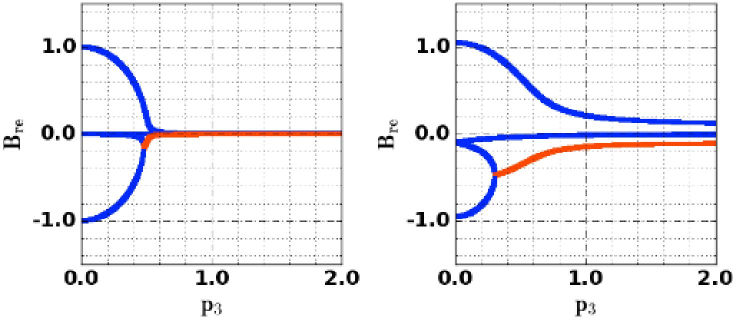
[17] Klähn, Fischer, Hempel, Astrophys.J. 836 (2017) 1, 8

Conclusions and remarks

Truncation[18]

$$\sout{g^2 D_{\rho\sigma}(p-q) = \delta_{\rho\sigma} \frac{1}{m_G^2} \Theta(\Lambda^2 - \vec{p}^2)}$$

$$g^2 D_{\rho\sigma}(p-q) = 3\pi^4 \eta^2 \delta^{\rho\sigma} \delta^{(4)}(p-q)$$



[18] Cierniak, Klähn, Acta Phys.Polon.Supp. 10 (2017) 811

Conclusions

- vBag is a model that introduces $D\chi SB$ and repulsive vector interactions into a standard Bag model.
- Vector interactions stiffen the quark EoS and help to achieve the 2 solar mass constraint for neutron stars.
- Standard NJL and BAG models can be derived by applying specific approximations to the quark DSE.
- Different sets of approximations to the quark DSE can produce a description of momentum dependent quarks which can be applied to astrophysical studies