Equation of State of a Magnetized Neutron System

Aric A. Hackebill   Efrain J. Ferrer

Eng. Science / Physics Dept.

City College of Staten Island, CUNY

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Outline

Motivation

How does an external magnetic field affect the thermodynamics of neutral many-particle systems?

- Prior investigation shows that the magnetic-field/magnetic-moment interaction has no significant influence on the EoS of a system of charged fermions:"E.J. Ferrer, V. de la Incera, D. Manreza Paret, A. Prez Martnez, and A. Sanchez Phys. Rev. D 91, 085041, 27 April 2015."
- Does an external magnetic field signficantly affect the EoS of neutral particle systems?

Model

Lagrangian density with $\vec{B} = (0, 0, B)$ and $k_N =$ "Neutron Anomalous Magnetic Moment"

$$L = \bar{\Psi}_N (i\gamma_\alpha \partial^\alpha - M_N + ik_N \sigma_{\alpha\nu} F^{\alpha\nu}) \Psi_N$$

Energy Spectrum with $\eta, \sigma = \pm 1$

$$E_{\eta,\sigma} = \eta \sqrt{{p_3}^2 + \left(\sqrt{{M_N}^2 + {p_1}^2 + {p_2}^2} + \sigma k_N B \right)^2}$$

Many-Particle Effective Lagrangian Density

$$L_E = \bar{\Psi}_N (i\gamma_\alpha \partial^\alpha + \gamma^0 \mu - M_N + ik_N \sigma_{\alpha\nu} F^{\alpha\nu}) \Psi_N$$

Thermodynamic Potential

The one loop Grand Canonical Potential with $p_0 = ip_4, p_4^* = ip_4 - \mu$ and $p_i^* = p_i$:

$$\Omega_N = \frac{1}{\beta} Tr\Big[ln[Z]\Big] = \frac{-1}{\beta} \sum_{p_4} \int \frac{d^3p}{(2\pi)^3} ln[Det(-\gamma^\alpha p_\alpha^* - M_N - ik_N B\gamma_2\gamma_1)]$$

We may write: $\Omega_N = \Omega_{vac} + \Omega_\beta$

$$\Omega_{vac} = -\int_{-\infty}^{\infty} \frac{d^3p}{(2\pi)^3} (E_{+,-} + E_{+,+})$$

$$\Omega_\beta = -\int_{-\infty}^{\infty} \frac{d^3p}{(2\pi)^3} \Big(\frac{1}{\beta} \sum_\sigma \big[ln[1 + e^{-\beta(E_{+,\sigma}+\mu)}] + ln[1 + e^{-\beta(E_{+,\sigma}-\mu)}]\big] \Big)$$

Many-Particle Thermodynamic Potential

In the zero temperature limit: $\Omega_\mu = \lim_{\beta \to \infty} \Omega_\beta$

$$\Omega_\mu = -\int_{-\infty}^{\infty} \frac{d^3 p}{(2\pi)^3} \left[(\mu - E_{+,-}) \Theta(\mu - E_{+,-}) + (\mu - E_{+,+}) \Theta(\mu - E_{+,+}) \right]$$

Many-Particle Potential Cont.

$$48\pi^2\Omega_\mu = \Bigg[2\left(\sqrt{1-\left(\frac{M_N+k_NB}{\mu}\right)^2}+\sqrt{1-\left(\frac{M_N-k_NB}{\mu}\right)^2}\right)\mu^4$$

$$+4\mathrm{k}_NB\left(sin^{-1}\left(\frac{M_N+K_NB}{\mu}\right)-sin^{-1}\left(\frac{M_N-K_NB}{\mu}\right)\right)\mu^3$$

$$+(8\mathrm{k}_NB(M_N+k_NB)-5(M_N+k_NB)^2)\sqrt{1-\left(\frac{M_N+k_NB}{\mu}\right)^2}\mu^2$$

$$+(\text{-}8\mathrm{k}_NB(M_N-k_NB)-5(M_N-k_NB)^2)\sqrt{1-\left(\frac{M_N-k_NB}{\mu}\right)^2}\mu^2$$

$$+(\mathrm{M}_N+k_NB)^3(3M_N-k_NB)\left(ln\left[1+\sqrt{1-\left(\frac{M_N+k_NB}{\mu}\right)^2}\right]-ln\left[\left|\frac{M_N+k_NB}{\mu}\right|\right]\right)$$

$$+(\mathrm{M}_N-k_NB)^3(3M_N+k_NB)\left(ln\left[1+\sqrt{1-\left(\frac{M_N-k_NB}{\mu}\right)^2}\right]-ln\left[\frac{M_N-k_NB}{\mu}\right]\right)\Bigg]$$

Asymmetries at $B \neq 0 \quad \mu \neq 0$

EJF, V. de la Incera, J. Keith, L.Portillo and P.Springsteen, PRC 82 (2010) 065802

$$\frac{1}{\beta V}\left\langle \tau^{\mu\nu} \right\rangle = \Omega_B \eta^{\mu\nu} + (\mu N + TS) u^\mu u^\nu + BM \eta_\perp^{\mu\nu}$$

$$\Omega_B = \Omega + \frac{B^2}{2}, \quad \eta_\perp^{\mu\nu} = \hat{F}^{\mu\rho} \hat{F}^{\nu}_{\rho}, \quad M = -\frac{\partial \Omega_B}{\partial B} \quad yields:$$

$$\boldsymbol{\epsilon = \Omega_B - \mu \frac{\partial \Omega_B}{\partial \mu}}$$

$$\boldsymbol{p^{\|} = -\Omega_B} \quad \text{and} \quad \boldsymbol{p^{\perp} = -\Omega_B + B \frac{\partial \Omega_B}{\partial B}}$$

Thermodynamic Quantities

Magentization

$$M = -\frac{\partial \Omega_\mu}{\partial B}$$

Energy Density

$$\epsilon = \Omega_\mu - \mu \frac{\partial \Omega_\mu}{\partial \mu}$$

Parallel Pressure

$$p_\parallel = -\Omega_\mu - \frac{B^2}{2}$$

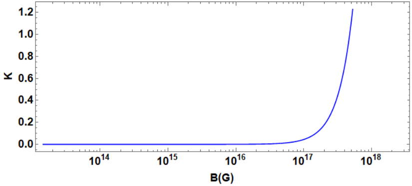
Perpendicular Pressure

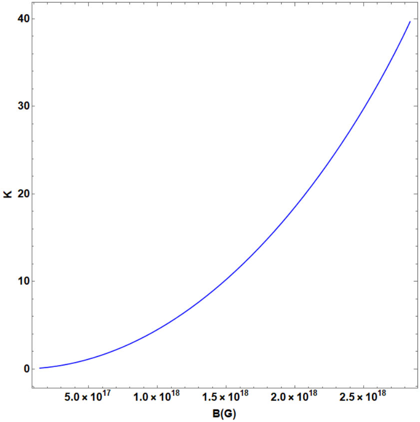
$$p_\perp = -\Omega_\mu - MB + \frac{B^2}{2}$$

Energy Density Ratio vs B

$$B_{max} = 2.8 \times 10^{18} G$$
$$B_{low} = 2.8 \times 10^{13} G$$

$$K = \frac{|ED(B) - ED(B_{low})|}{ED(B_{low})} \times 100\%$$

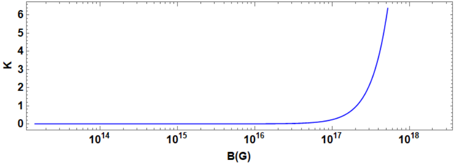


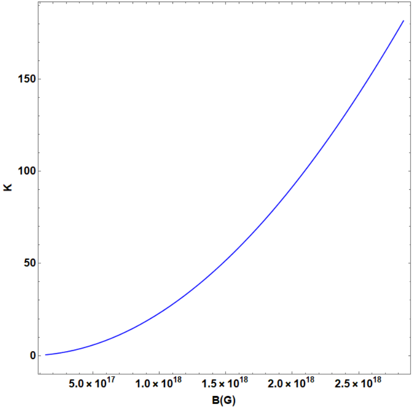


Parallel Pressure Ratio vs B

$$B_{max} = 2.8 \times 10^{18}\, G$$
$$B_{low} = 2.8 \times 10^{13}\, G$$

$$K = \frac{|Ppar(B) - Ppar(B_{low})|}{Ppar(B_{low})} \times 100\%$$

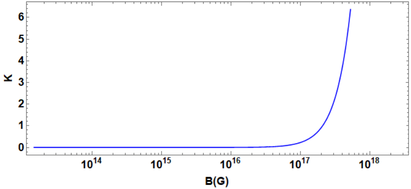


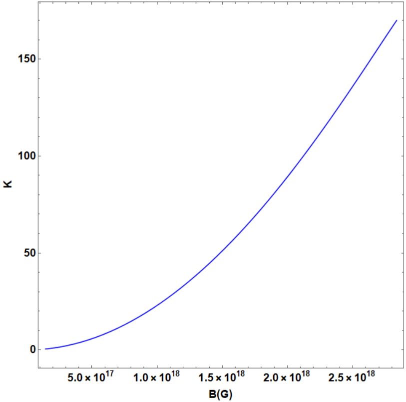


Perp Pressure Ratio vs B

$B_{max} = 2.8 \times 10^{18} G$
$B_{low} = 2.8 \times 10^{1} 3 G$

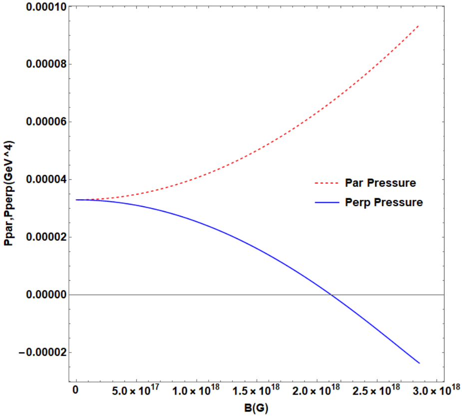
$$K = \frac{|Pperp(B) - Pperp(B_{low})|}{Pperp(B_{low})} \times 100\%$$

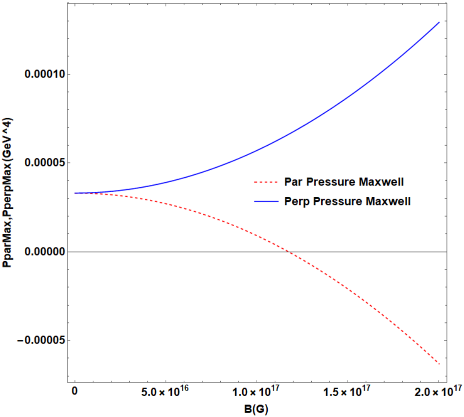




Pressure Splitting vs B

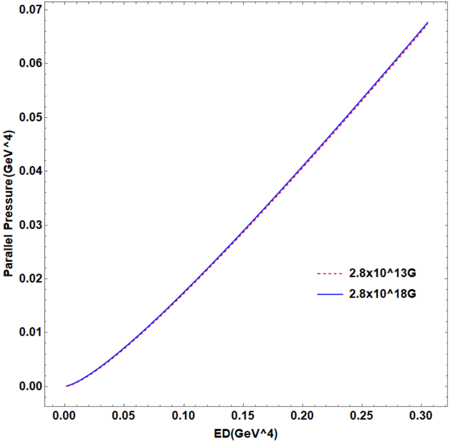
$$\mu = 1000\,MeV\,, \quad p_{\parallel} = -\Omega_\mu - \frac{B^2}{2}, \quad p_{\perp} = -\Omega_\mu - MB + \frac{B^2}{2}$$



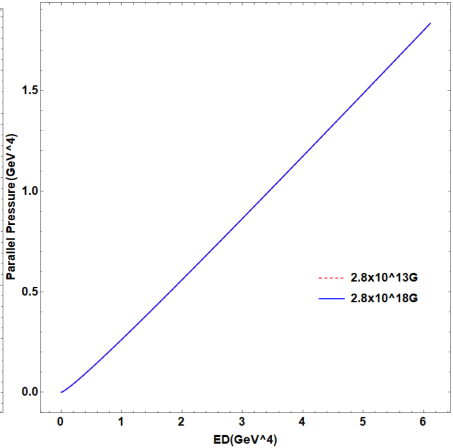


Parallel Pressure EoS Without Maxwell Term

$\mu = (1000MeV, 2000MeV)$

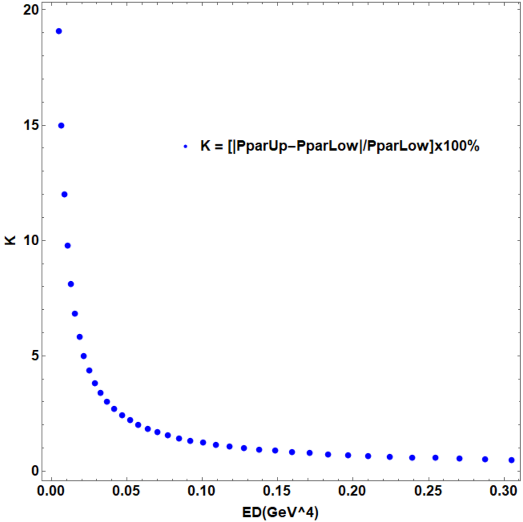


$\mu = (1000MeV, 4000MeV)$



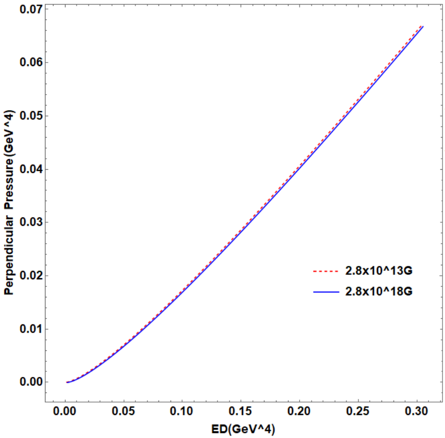
Parallel Pressure Ratio vs ED

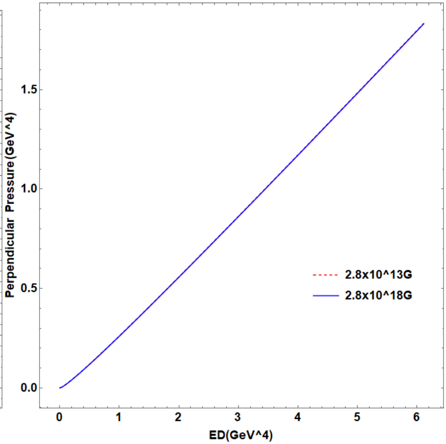
$\mu = (1000 MeV, 2000 MeV)$



Perp Pressure EoS Without Maxwell Term

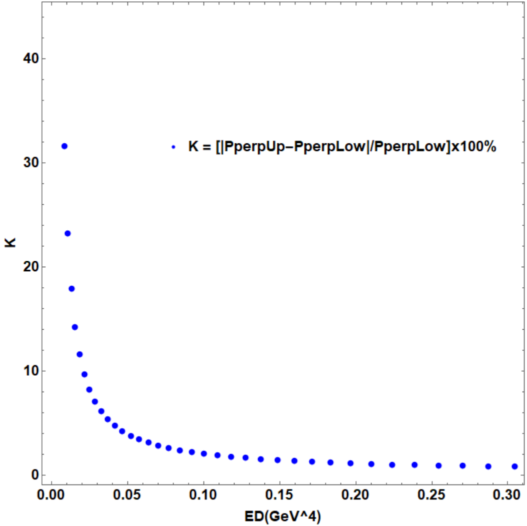
$\mu = (1000 MeV, 2000 MeV)$ $\mu = (1000 MeV, 4000 MeV)$





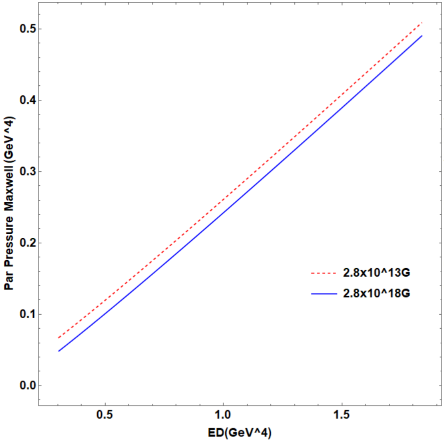
Perp Pressure Ratio vs ED

$\mu = (1000MeV, 2000MeV)$

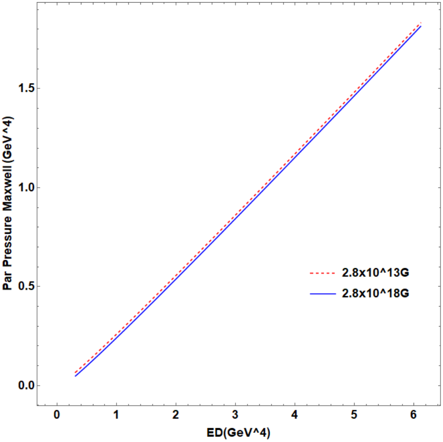


Parallel Pressure EoS with Maxwell Term

$\mu = (2000MeV, 3000MeV)$

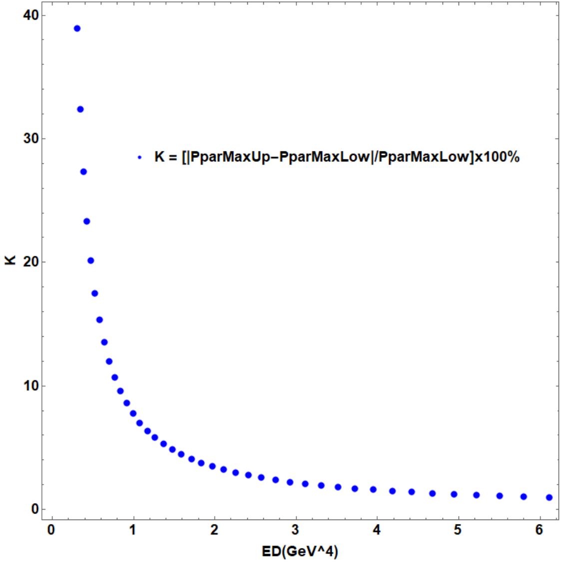


$\mu = (2000MeV, 4000MeV)$



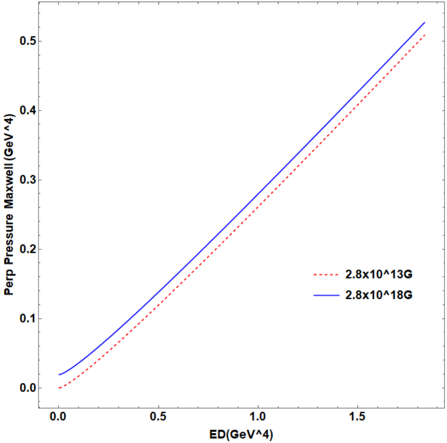
Maxwell Parallel Pressure Ratio vs ED

$\mu = (2000 MeV, 4000 MeV)$

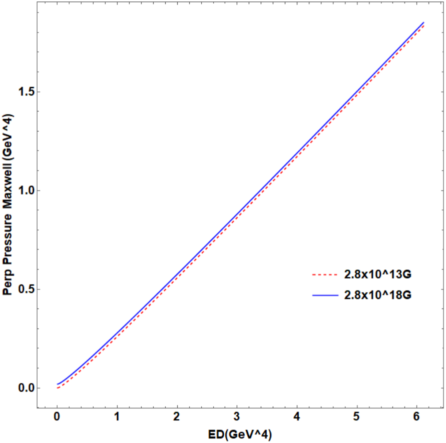


Perp Pressure EoS with Maxwell Term

$\mu = (1000 MeV, 3000 MeV)$

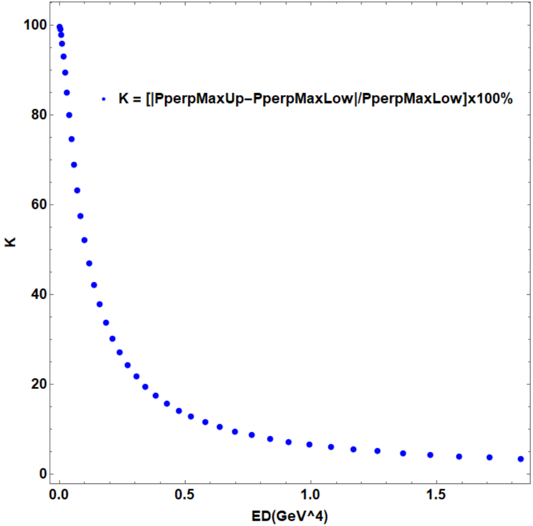


$\mu = (1000 MeV, 4000 MeV)$



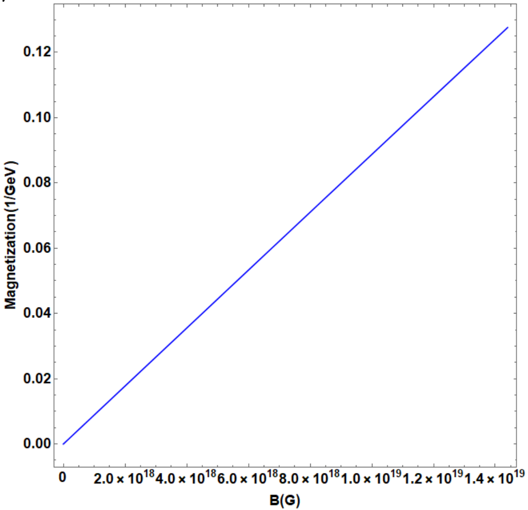
Maxwell Perp Pressure Ratio vs ED

$\mu = (1000MeV, 3000MeV)$



Magnetization vs B

$\mu = 1000 MeV$



Concluding Remarks

- The pressures and energy density only begin to vary at magnetic field strengths close to the maximum allowed value.
- There is a maximum field value that produces a zero pressure
- The pressure splitting is only significant at magnetic field values close to the maximum allowed field.
- On the domain considered, no significant change in the equations of state is observed.